Seat No.: ________ Enrolment No._______

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER • 2014

Subject Code: 110014 Date: 19-06-2014

Subject Name: Calculus

Time: 02:30 pm - 05:30 pm Total Marks: 70

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) State Euler's Theorem on homogeneous function. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$,then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u\cos 3u$ **05**

(b) If $u = x^2y + y^2z + z^2x$, prove that **05**

(i) $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = (x+y+z)^2$ (ii) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = 6(x+y+z)$

(c) $Evaluate\,(i)\ \lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2y}{x^4+y^2}$ $(ii)\ \lim_{\substack{x\to 0\\ y\to 0}} \frac{x^3-y^3}{x^2+y^2}$ **04**

Q.2 (a) Discuss the maxima and minima of $f(x,y) = x^3 + y^3 - 3xy$ **05**

(b) Expand $f(x,y) = x - 20y - 4x^2 + xy + 6y^2 + 20$ about the point $(-1,2)$ in Taylor's series by obtaining first three terms only. **05**

(c) If $x = e^v\sec u$ *and* $y = e^v\tan u$ then in usual notations find the values of J and J' **04**

Q.3 (a) Using Maclaurin's expansion, expand (i) $(1+x)^{\frac{1}{x}}$ upto the term x^2 **05**

(ii) $\log_e \sec x$ *in powers of* x

(b) Evaluate (i) $\lim_{x\to 0}\frac{(1+x)^{\frac{1}{x}} - e + \frac{ex}{2}}{x^2}$ (ii) $\lim_{x\to 0}\frac{x-\tan x}{x^3}$ **05**

(c) Using Taylor's series, expand $\log_e x$ in powers of $(x-1)$ and hence find the value of $\log_e 1.1$ **04**

Q.4 (a) Trace the curve $y^2(2a-x) = x^3$. Also draw the rough sketch. **05**

(b) Using Reduction formulae, evaluate **05**

(i) $\int_0^{\pi} x\sin^7 x\cos^6 x\,dx$ (ii) $\int_0^{\infty}\frac{x^2}{(1+x^2)^{7/2}}\,dx$

(c) Do as directed (each question is of one mark) **04**

(i) Find the point of inflexion on the curve $f(x) = 2(x-1)^3$

(ii) Find the node of the curve $x^3 + y^3 - 3xy = 0$

(iii) The curve $r = a(1+\cos\theta)$ is symmetric about ....

(iv) Find the linearization of $f(x) = 2\sin x + 3\cos x\ at\ x = 0$

Q.5 **(a)** Test the convergence of **05**

(i) $\sum_{n=1}^{\infty} \frac{3n^2+5n}{7+n^4}$ (ii) $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\ldots\ldots, where\ x>0$

(b) Discuss the convergence of **05**

(i) $1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots$ (ii) $\sum_{n=1}^{\infty}\left(\frac{n+1}{n+2}\right)^n x^n\ ,\ x>0$

(c) Test the convergence of (i) $\int_0^{\infty}\frac{1}{1+x^2}\,dx$ (ii) $\int_0^{\infty}\frac{\sin ax}{x}\,dx$ **04**

Q.6 **(a)** By changing to polar coordinates, evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dxdy$. Also sketch the regions. **05**

(b) Evaluate $\iint y\,dxdy$ over the region bounded by $y=x^2$, $x=0, x+y=2$ in the first quadrant. **05**

(c) Evaluate (i) $\int_0^1\int_0^1\frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$ (ii) $\int_0^{\pi}\int_0^{1-\cos\theta} r^2\sin\theta\,dr\,d\theta$ **04**

Q.7 **(a)** A loop of the curve $y^2=x^2(1-x^2)$ is rotated about the Y-axis. Find the volume generated. **05**

(b) Find by double integration the area outside the circle $r=a$ and inside the cardioid $r=a(1+\cos\theta)$ **05**

(c) Derive reduction formula for $\int_0^{\frac{\pi}{2}}\sin^n x\,dx\ \ where\ n\geq 2\,, n\in N$ **04**

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