

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 110014

Date: 29-12-2014

Subject Name: Calculus

Time: 10:30 am - 01:30 pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a)(i) Evaluate 03

(1) $\lim_{x \rightarrow 0} \frac{2x - x \cos x - \sin x}{x^3}$

(2) $\lim_{x \rightarrow 0} \sin x^{\tan x}$

(ii) Using Taylor's series Evaluate $\sin 44^\circ$ 03

(iii) Evaluate $\int_0^a x^{3/2} \sqrt{a-x} dx$ 02

(b)(i) Find the equation of tangent plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at $(2, 1, -3)$. 03

(ii) Expand $3x^3 + 8x^2 + x - 2$ in powers of $x - 3$. 03

Q.2 (a)(i) Test the convergence of $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. 03

(ii) Find the volume of the solid generated by the revolution about the initial line of the cardioids $r = a(1 - \cos \theta)$. 04

(b)(i) Discuss the continuity of 03

$$f(x, y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

(ii) If $u = x^3y + e^{xy^2}$ then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ 04

Q.3 (a)(i) Find the interval on which the function $f(x) = x^4 - 4x^3 + 10$ is increasing and decreasing. **03**

(ii) Trace the curve $r = a(1 + \cos \theta)$, $a > 0$. **04**

(b)(i) Evaluate $\int_0^{\pi} \sin^2 \theta (1 + \cos \theta)^4 d\theta$ **03**

(ii) Find the maximum and minimum value of $y = x^3 - 9x^2 + 24x + 2$ **04**

Q.4 (a)(i) Evaluate $\int_0^1 \int_0^{\sqrt{x}} (x^2 + y^2) dy dx$ **03**

(ii) Change the order of integration and evaluate $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dx dy$ **04**

(b)(i) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} (x^2 + y^2) dy dx$ by changing into polar coordinates. **03**

(ii) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz dz dy dx$ **04**

Q.5 (a)(i) Evaluate (1) $\lim_{n \rightarrow \infty} \sqrt[n]{n}$ (2) $\lim_{n \rightarrow \infty} \frac{\sin^2 n}{n}$ **03**

(ii) Examine the convergence of **04**

$$(1) \sum_{n=1}^{\infty} \frac{2n^2 + 3n}{5 + n^5} \quad (2) \sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$$

(b)(i) The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and the y -axis is revolved about x -axis then find the volume of the solid generated. **03**

(ii) Test the convergence of $x + \frac{x^2}{2^3} + \frac{x^3}{3^3} + \dots$ **04**

Q.6 (a)(i) If $u = r^m$ then prove that **03**

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$$

(ii) If $z = f(x, y)$ and if $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ then prove that **04**

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

(b)(i)

Expand $\tan^{-1}\left(\frac{y}{x}\right)$ in powers of $(x-1)$ and $(y-1)$ using Taylor's

03

(ii)

If $u = x^3 y^2 \sin^{-1}\left(\frac{y}{x}\right)$ then show that

04

$$(1) \quad x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 5u$$

$$(2) \quad x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 20u$$

Q.7 (a)(i) Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1, 1, 1)$.

03

(ii)

if $u = \sin^{-1} \left[\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{4}} - y^{\frac{1}{4}}} \right]$ then prove that

04

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{12} \tan u$$

(b)(i) Evaluate $\iiint z(x^2 + y^2) dv$ over the volume of the cylinder

03

$x^2 + y^2 = 1$ intercepted by the planes $z = 2$ and $z = 3$.

(ii)

Find $\frac{dy}{dx}$ if (1) $x^2 + y^3 = 7xy$ (2) $(\cos x)^y = x^{\sin y}$

04
