Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER 1st / 2nd (OLD) EXAMINATION WINTER 2016

Subject Code: 110014 **Date: 24/01/2017**

Subject Name: Calculus

Time:10:30 AM TO 1:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)(1)** Test the convergence of the sequence $\{2-(-1)^n a\}$ **03**

(2) Show that the sequence $\{u_n\}$ whose n[th] term is $u_n = \frac{1}{1!}+\frac{1}{2!}+\ldots+\frac{1}{n!}$ *where* $n \in N$ is monotonic increasing and bounded. Is it convergent? **04**

(b) Expand $\tan^{-1}(x+h)$ in power of h and hence, find the value of $\tan^{-1}(1.003)$ up to 5 places of decimal. **07**

Q.2 **(a)(1)** Find the expansion of $\log(1+x)$ **03**

(2) Determine whether the following series converge or diverge. Find the sum of the series if it converges. $\sum_{n=1}^{\infty}\left[\tan^{-1} n - \tan^{-1}(n+1)\right]$ **04**

(b) Test the convergence of the series $\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}+\ldots$ **07**

Q.3 **(a)(1)** Evaluate : $\int_{-\infty}^{\infty}\frac{1}{1+x^2}dx$ **03**

(2) Find the linearization of $f(x,y,z) = x^2 - xy + 3\sin z$ at the point (2,1,0) **04**

(b) Trace the curve $y^2(a+x) = x^2(b-x)$ **07**

Q.4 **(a)(1)** If $z = e^{xy}, x = t\cos t, y = t\sin t$, find $\frac{dz}{dt}$ at $t = \frac{\pi}{2}$ **03**

(2) Find the equation of tangent plane and normal line to the surface $x^2yz + 3y^2 = 2xz^2 - 8z$ at the point (1,2,-1) . **04**

(b) Change the order of integration and hence evaluate $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\ dydx$ **07**

Q.5 **(a)(1)** Find $\lim_{(x,y)\to(0,0)}\frac{xy}{y^2-x^2}$ **03**

(2) If $u = \tan^{-1}\left(\frac{x}{y}\right)$ where $x^2 + y^2 = a^2$ find $\frac{du}{dx}$. **04**

(b) State Euler's theorem. If $z = x^n f\left(\frac{y}{x}\right) + y^{-n} f\left(\frac{x}{y}\right)$ prove that **07**

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\partial y} + y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = n^2 z$$

Q.6 **(a)** Find the maxima and minima of the function $x^3 + y^3 - 63(x + y) + 12xy$ **07**

(b) Evaluate $\iint (x^2 + y^2)\,dxdy$ over the region bounded by the lines y=4x, x+y=3, y=0, y=2. **07**

Q.7 **(a)** Evaluate $\iiint z^2\,dxdydz$ over the region common to the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 2x$ **07**

(b) Using the method of cylindrical shell, find the volume of the solid obtained by rotating about the x-axis the region under the curve $y = \sqrt{x}$ from 0 to 1. **07**

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