Seat No.: ________ *Enrolment No.*_______

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER • 2014

Subject Code: 2110014 **Date: 19-06-2014**
Subject Name: Caculus
Time: 02:30 pm - 05:30 pm **Total Marks: 70**

Instructions:
1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) **1** The value of $\lim_{x \to 0} x sin\frac{1}{x}$ is **07**
(a) 1 (b) π (c) 0 (d) ∞

2 $y = \log(sinx)$ then value of $\frac{d^2y}{dx^2} =$
(a) $sec^2 x$ (b) $-cosec^2 x$ (c) $-cosec\, xcotx$ (d) $secxtanx$

3 Curve : $x = t^2 - 1, y = t^2 - t$.For which value of t tangents of the curve are parallel to the x-axis?
(a) $t = 0$ (b) $t = \frac{1}{\sqrt{3}}$ (c) $t = \frac{1}{2}$ (d) $t = -\frac{1}{\sqrt{3}}$

4 $\int_{-1}^{1} x|x|\ dx =$ …………..
(a) 2 (b) 1 (c) 0 (d) $none\ of\ these$

5 If $f(x) = 4 - 2x, x < 1$
$= 6x - 4, x \geq 1$ then find $\lim_{x \to 1} f(x)$
(a) -2 (b) 2 (c) 0 (d) $doesn't exist.$

6 If $f: R \to R, f(x) = 3x + 2,\ then\ find\ f^{-1}$
(a) not possible (b) $3x - 2$ (c) $2x - 3$ (d) $\frac{x-2}{3}$

7 What is the solution for following equations ?
$2x + 3y - 5 = 0, 4x + 6y - 7 = 0.$
(a) no solution (b) infinitely many solution
(c) unique solution (d) $\{(0,0), (-5,-7)\}$

(b) **1** $\lim_{n \to \infty} \frac{1-n^2}{\sum n} =$ **07**
(a) -2 (b) 1 (c) 2 (d) $doesn't exist.$

2 $\int \frac{1}{\sqrt{x}} cos\sqrt{x}\, dx = \cdots + c$
(a) $\frac{2}{3}cos^{\frac{3}{2}}x$ (b) $2\sqrt{x}$ (c) $2sin\sqrt{x}$ (d) $\frac{2}{3}cos\sqrt{x}.$

3 Find the area of the curve $y = x^2 + 1$ bounded by x-axis and lines $x = 1\ and\ x = 2.$
(a) $\frac{3}{10}$ (b) $\frac{10}{3}$ (c) 6 (d) $\frac{1}{6}$

4 $f(x) = [x], x \in R$ then f(x) is

(a) Continuous for all real numbers.
(b) Continuous for all integers
(c) discontinuous for all integers
(d) none of above.

5 Find the vertical asymptotes of the curve $y = \frac{2x^2}{x^2-1}$.
(a) $x = 1$ (b) $x = -1$ (c) $x = \pm 1$ (d) $y = 2$

6 $f(x) = x^3 - 3x^2 + 3x - 100$ is … …….. function.
(a)Increasing (b) decreasing (c) constants (d) none

7 Find the value of c using Lagrange's mean value theorem for
$f(x) = e^x, x \in [0,1]$
(a) $log 1$ (b) $\log c$ (c) $\log(1-e)$ (d) $\log(e-1)$

Q.2 **(a)** **(1)** Evaluate $\lim_{x\to 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$ **02**

(2) Test for convergence the series **02**
$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots\dots\dots$

(3) Expand $\log \sin x$ in powers of $(x-2)$ **03**

(b) **(1)** Test for convergence the series **04**
$2 + \frac{3}{2}x + \frac{4}{3}x^2 + \frac{5}{4}x^3 + \dots\dots\dots$

(2) Trace the curve $r = a \sin 3\theta$ **03**

Q.3 **(a)** **(1)** (i) Evaluate $\lim_{x\to 0}(a^x + x)^{\frac{1}{x}}$ **04**
(ii) Test the convergence of $\sum_{n=0}^{\infty} \frac{3^{2n}}{2^{3n}}$.

(2) Trace the curve $x^3 + y^3 = 3axy$. **03**

(b) **(1)** Test the convergence of $\sum_{n=1}^{\infty} \frac{[(n+1)x]^n}{n^{n+1}}$. **04**

(2) Evaluate improper integral $\int_0^3 \frac{1}{\sqrt{3-x}}\,dx$. **03**

Q.4 **(a)** **(1)** (i) If $u = \ln\left(\frac{x^7 + y^7 + z^7}{x+y+z}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 6$. **04**
(ii) State modified Euler's theorem and find the degree of homogeneous function $u(x,y) = \frac{1}{x^2} + \frac{1}{xy} + \frac{\log x - \log y}{x^2}$.

(2) $z = f(x,y), x = e^u + e^{-v}, y = e^{-u} - e^v$ then prove that **03**
$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial u}{\partial y}$.

(b) **(1)** $If u = e^{3xyz}$ show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (3 + 27xyz + 27x^2y^2z^2)\, e^{3xyz}$. **04**

(2) Show that $f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}; (x,y) \neq (0,0)$ **03**
$= 0 \quad , (x,y) = (0,0)$
is continuous at the origin.

Q.5 **(a)** **(1)** Find the extreme values of the function **04**
$f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

(2) Show that the surfaces $z = xy - 2$ and $x^2 + y^2 + z^2 = 3$ have the same tangent plane at (1,1,1). **03**

(b) **(1)** Find the minimum value of $x^2\, y\, z^3$ subject to the condition **04**
$2x + y + 3z = a$ using Lagrange's method of undetermined multipliers.

(2) Expand $\sin xy$ in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$ upto second degree terms. **03**

Q.6 **(a)** **(1)** Sketch the region of integration, reverse the order of integration and evaluate $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y} dydx.$ **04**

(2) Evaluate the integral $\int_0^{\frac{\pi}{2}} \int_0^{1-sin\theta} r^2 \; cos\theta drd\theta.$ **03**

(b) **(1)** Evaluate the integral $\int_0^2 \int_1^2 \int_0^{yz} xyz \, dxdydz.$ **04**

(2) Evaluate the integral $\int_0^a \int_0^{\sqrt{a^2-y^2}} y^2 \; \sqrt{x^2+y^2} dydx$.by changing into polar co-ordinates. **03**

Q.7 **(a)** **(1)** Find the volume bounded by the cylinder $x^2+y^2=4$ and the planes $y+z=3, z=0$. **04**

(2) Evaluate $\iint_R \; x^2+y^2 dA$ by changing the variables ,where R is the region lying in first quadrant and bounded by the hyperbola $x^2-y^2=1, x^2-y^2=9, xy=2and \; xy=4$. **03**

(b) **1** The region between the curve $y=\sqrt{x}, 0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid.Find its volume. **04**

2 Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$. **03**

****