

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER- 1st / 2nd EXAMINATION (New Syllabus) – WINTER 2014

Subject Code: 2110014

Date: 29-12-2014

Subject Name: Calculus

Time: 10:30 am - 01:30 pm

Total Marks: 70

Instructions:

1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Objective Question

(a) Answer the following questions with most appropriate answer

07

1. The maximum value of $y = 3 \cos 2x$ is ____
(a) 1 (b) 2 (c) -1 (d) 3
2. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then dy/dx ____
(a) $\sqrt{y/a}$ (b) $\sqrt{x/a}$ (c) $-\sqrt{y/x}$ (d) $-\sqrt{y/a}$
3. The $(a-x)y^2 = x^2(a+x)$ is symmetric about
(a) X-axis (b) Y-axis (c) both X and Y axis (d) line $Y = X$
4. Tangents at origin to the curve $y^2(a+x) = x^2(3a-x)$ is ____
(a) $\pm \sqrt{3}x$ (b) 1 (c) $\pm \sqrt{2}x$ (d) none of these
5. The parametric equations of $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ ($a \neq 0$) are
(a) $x = a \cos \theta, y = a \sin \theta$ (b) $x = a^3 \cos \theta, y = a^3 \sin \theta$
(c) $x = a \cos^3 \theta, y = a \sin^3 \theta$ (d) $x=1, y=0$
6. The area bounded by the curve $y = x^3$, x-axis and two ordinates $x=1$ to $x=2$ equal to
(a) 15/2 (b) 15/3 (c) 15/5 (d) 15/4
7. $\lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4} =$ ____
(a) 7/4 (b) 3/4 (c) 2 (d) -3/4

(b) Answer the following questions with most appropriate answer

07

1. The slope of the tangent to the curve $y = xe^x$ at $(0,0)$ is ____
(a) 0 (b) 1 (c) -1 (d) 4
2. The point of inflection of $\frac{x^3}{3} - \frac{x^2}{2} - 2x + 14$ is ____
(a) 1/2 (b) -1/2 (c) -1 (d) -2
3. Using the matrix method, the solution of $x + y = 2, 4x + y = 6$,
(a) $\left(\frac{-4}{3}, \frac{2}{3}\right)$ (b) $\left(\frac{4}{3}, \frac{-2}{3}\right)$ (c) $\left(\frac{4}{3}, \frac{2}{3}\right)$ (d) (1,2)
4. $\int e^{2x} \cos 3x \, dx =$ ____ + c
(a) $\frac{e^{2x}}{13}(2 \cos 3x + 3 \sin 3x)$ (b) $\frac{e^{2x}}{13}(3 \cos 3x + 2 \sin 3x)$
(c) $\frac{e^{2x}}{13}(2 \cos 3x - 3 \sin 3x)$ (d) none of these
5. If $y = \ln(\sqrt{2}x)$ the derivative of the function y with respect to x is ____
(a) 0 (b) -1/2x (c) 1/2x (d) 1/x
6. $f(x) = x^2 + 4x + 5$ has the minimum value ____

- (a) 0 (b) 1 (c) -1 (d) 2
7. A curve which passes through the origin and has the slope $-1/3$ is given by
 (a) $x + 3y - 1 = 0$ (b) $x + 3y = 0$ (c) $x - 3y = 0$ (d) none of these

OR

Q:1 (a) Answer the given MCQ.

07

- $F(x)$ is strictly increasing function on $\mathbb{R}$ then_____
 - $f'(x) = 0$ for all x
 - $f'(x) > 0$ for all x
 - $f'(x) < 0$ for all x
 - none of these
- $F(x)$ is strictly decreasing function on $\mathbb{R}$ then_____
 - $f'(x) = 0$ for all x
 - $f'(x) > 0$ for all x
 - $f'(x) < 0$ for all x
 - none of these
- $dy/dx = ky$, $k > 0$ is the differential equation for_____
 - Population Model
 - Mixing problem model
 - Cooling Model
 - none of these
- $dy/dx = x$ then $y =$ _____
 - $y = x^2/2 + C$
 - $y = x^2/2 - C$
 - $y = x^2/2$
 - none of these
- Curve of $y = x^2 + 3$ is_____
 - Symmetric with respect to x axis.
 - Symmetric with respect to y axis.
 - Symmetric with respect to origin.
 - none of these
- $r = a \cos \theta$ is _____
 - A circle
 - A line
 - A parabola
 - None of these
- $\int_0^1 x^2 dx$ is _____
 - $1/3$
 - $2/3$
 - $1/2$
 - none of these

(b) Answer the given MCQ.

07

- $\int_{-a}^a f(x) dx = 0$ if _____
 - f is an odd function
 - f is neither odd nor even function
 - f is an even function
 - none of these
- $z = x^2 + y^2$ is _____
 - Cone
 - A paraboloid
 - A cylinder
 - none of these
- $\frac{\partial}{\partial x} (x^2 + y^2) =$ _____
 - 2
 - 1
 - 0
 - 1
- If $z = x^2 - y^2$ then $\frac{\partial^2 z}{\partial x^2} =$ _____
 - 2y
 - 0
 - 2z
 - none of these
- Equation of tangent plane of $z = x$ at $(2, 0, 2)$ is _____
 - $x - 2 = 0$
 - $x - 1 - y - z = 0$
 - $x + 1 - y - z = 0$
 - $x - 1 - y - z = 2$
- If $z = x^2 + y^2 + 3$ minimum value z is _____

A)

C)

B)

D)

7. $Y = \sin 2x$ is increasing in interval _____

A) $(0, \frac{\pi}{2})$

C) $(0, \frac{\pi}{2})$

B) $(0, \frac{\pi}{4})$

D) None of these

Q.2 (a) Test the convergence of $\frac{1}{1 \cdot 2 \cdot 3} + \frac{3}{2 \cdot 3 \cdot 4} + \frac{5}{3 \cdot 4 \cdot 5} + \dots$ **03**

(b) Prove that $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \frac{16}{81} + \dots$ converges and find its sum. **04**

(c) State Taylor's series for one variable and hence find $\sqrt{36.12}$ **07**

Q.3 (a) Find $\frac{\partial w}{\partial r}, \frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$ where $x = \frac{r}{s}, y = r^2 + \ln s, z = 2r$ **03**

(b) Find $\frac{dy}{dx}$ when $(\cos x)^y = (\sin y)^x$ **04**

(c) If $u = \operatorname{cosec}^{-1} \left[\frac{x^{\frac{1}{2}} + y^{\frac{1}{2}}}{x^{\frac{1}{3}} + y^{\frac{1}{3}}} \right]^{\frac{1}{2}}$ **07**

show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144} (13 + \tan^2 u)$. Also state Euler's modified theorem.

Q.4 (a) Evaluate $\lim_{x \rightarrow 0} \left[\frac{1}{x^2} - \frac{1}{\sin^2 x} \right]$ **03**

(b) Expand $e^x \sin y$ in powers of x and y upto third degree. **04**

(c) A rectangular box, open at the top, is to have a volume 32 c.c. Find the dimensions of the box requiring least material for its construction. **07**

Q.5 (a) Find the volume of the tetrahedron bounded by the plane $x + y + z = 2$ and the planes $x = 0, y = 0$ and $z = 0$. **03**

(b) Trace the curve $r = a(1 + \cos \theta)$ **04**

(c) Evaluate $\int_0^{4a} \int_{x^2/4a}^{2\sqrt{ax}} dy dx$ by changing the order of integration. **07**

Q.6 (a) Evaluate $\int_0^\infty \frac{dx}{x^2 + 1}$ **03**

(b) Evaluate the integral $\int_0^1 \int_0^{1-x} e^{\frac{y}{x+y}} dy dx$ by changing the variables $x + y = u, y = uv$ **04**

- (c) Evaluate $\iint_R x^2 \, dA$, where R is the region in the first quadrant bounded by the hyperbola $xy = 16$ and the lines $y = x$, $y = 0$ and $x = 8$.

07

- Q.7 (a) Test the convergence of $\sum_{n=1}^{\infty} \frac{2 \tan^{-1} n}{1 + n^2}$

03

- (b) Find the equation of tangent plane and normal line to the surface $xyz = 6$ at (1,2,3)

04

- (c) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz \, dz dy dx$

07
