Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER 1st / 2nd (NEW) EXAMINATION WINTER 2016

Subject Code: 2110014 **Date: 24/01/2017**

Subject Name: Calculus

Time:10:30 AM TO 1:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

		Mark
Q.1	**Objective Question (MCQ)**	
(a)		**07**

1. The sequence $\sin\left(\frac{\pi}{6}+\frac{1}{n}\right)$ converges to
(a) 0 (b) 1 (c) -1 (d) 0.5
2. The sum of the series $\sum_{n=1}^{\infty}\left(\frac{a}{b}\right)^{n}$ is ---
(a) $\frac{b}{b-a}$ (b) $\frac{a}{b-a}$ (c) $\frac{b}{a-b}$ (d) $\frac{a}{b}$
3. The value of the limit $\lim_{x\to 6}\frac{\sin(x-6)}{x-6}=$ ---
(a) 0 (b) 1 (c) -1 (d) 0.5
4. Asymptote parallel to Y-axis of the curve $y=\frac{x^2}{x-3}$ is the line ---
(a) x = 0 (b) y = 3 (c) x = 3 (d) not exist
5. Let $f(x,y)=y\sin(xy)$. The value of $f_x(\pi,1)$ is ---
(a) 0 (b) 1 (c) -1 (d) 2.5
6. $\int_1^2\int_1^2\frac{1}{xy}\,dx\,dy$ = ---
(a) 0 (b) $(\log 2)^2$ (c) 1 (d) $\log 2$
7. The coefficient of x^5 in the expansion of e^x is ---
(a) $\frac{1}{5}$ (b) $\frac{1}{4!}$ (c) $\frac{1}{5!}$ (d) 5

(b) **07**

1. $\sum_{n=1}^{\infty}\frac{2n}{2n-1}$ is ---
(a) convergent and sum is 0 (b) convergent and sum is 1
(c) divergent (d) oscillating
2. If $f=e^{x/y}\tan\left(\frac{x}{y}\right)+e^{y/x}\cot\left(\frac{y}{x}\right)$, then value of $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$ is ---
(a) 0 (b) f (c) $-f$ (d) $2f$
3. The curve $x^3y+y^3x=3$ is symmetric about the ---
(a) X-axis (b) Y-axis (c) origin (d) line y = x
4. What does the region of $\int_1^4\int_2^6 dx\,dy$ represents?
(a) rectangle (b) square (c) circle (d) triangle
5. The value of $\int_0^{\infty}\frac{1}{x^2+1}\,dx$ = ---
(a) π (b) $\frac{\pi}{2}$ (c) 0 (d) 1
6. The minimum value of $f(x,y)=x^2+y^2$ is ---
(a) 1 (b) 2 (c) 4 (d) 0
7. If $x=u+3v,\ y=v-u$ then value of $J=\frac{\partial(x,y)}{\partial(u,v)}$ is ---
(a) -1 (b) 4 (c) 5 (d) 7

Q.2	**(a)**	You drop a ball from *a* meters above a flat surface. Each time the ball hits the surface after falling a distance *h*, it rebounds a distance *rh*, where $0 < r < 1$. Find the total distance ball travels up and down when $a = 6$ m and $r = 2/3$ m.	**03**
	(b)	Evaluate (1) $\lim_{x\to 0}\frac{e^x+e^{-x}-x^2-2}{\sin^2 x-x^2}$ (2) $\lim_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\log x}\right)$	**04**
	(c)	Obtain the Maclaurin's series of $\log_e(1+x)$ and hence find the series of $\log_e\left(\frac{1+x}{1-x}\right)$ and then obtain approximate value of $\log_e\left(\frac{11}{9}\right)$.	**07**
Q.3	**(a)**	Show that $f(x,y)=\begin{cases}\frac{2x^2y}{x^2+y^2}, & (x,y)\neq 0\\ 0 & (x,y)=0\end{cases}$ is not continuous at the origin.	**03**
	(b)	If $\theta = t^n e^{-r^2/4t}$, then find the value of n for which $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}$.	**.04**
	(c)	State Euler's theorem on homogenous function of two variables. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin^3 u\cos u$.	**07**
Q.4	**(a)**	Find the Jacobian of the transformation $x=\rho\sin\phi\cos\theta,\ y=\rho\sin\phi\sin\theta,\ z=\rho\cos\phi$.	**03**
	(b)	Find the tangent plane and normal line of the surface $f(x,y,z)=x^2+y^2+z-9=0$ at the point P(1, 2, 4).	**04**
	(c)	A rectangular box open at the top is to have a volume of 32 cubic units. Find the dimensions of the box requiring least material for its construction.	**07**
Q.5	**(a)**	Evaluate $\int_{-1}^{1}\int_{0}^{2}(1-6x^2y)\,dx\,dy$.	**03**
	(b)	Evaluate $\int_0^1\int_0^{\sqrt{z}}\int_0^{2\pi}(r^2\cos^2\theta+z^2)\,r\,d\theta\,dr\,dz$.	**04**
	(c)	Change the order of integration and evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\frac{e^y}{(e^y+1)\sqrt{1-x^2-y^2}}\,dy\,dx$	**07**
Q.6	**(a)**	Let $S=\sum_{n=1}^{\infty}nx^n$ where $\lvert x\rvert<1$. Find the value of x in (0, 1) such that $S=2x$.	**03**
	(b)	Check for convergence/divergence (1) $\sum_{n=1}^{\infty}\frac{2n^2-3n}{n^2(n-2)(n^2+5)}$ (2) $\sum_{n=1}^{\infty}\frac{2^n}{n^2+1}$	**04**
	(c)	(1) Check for absolute/conditional convergence of $\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}+\sqrt{n+1}}$. (2) For the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{2n-1}}{2n-1}$, find the radius and interval of convergence.	**03** **04**

Q.7 **(a)** The graph of $y = x^2$ between x = 1 and x = 2 is rotated around the X-axis. Find the volume of a solid so generated. **03**

(b) Test the convergence of the improper integrals. If convergent then evaluate the same. **04**

(1) $\int_0^1 \frac{dx}{1-x}$ (2) $\int_0^{\infty} \frac{1}{(1+x^2)(1+\tan^{-1}x)}\,dx$

(c) Trace the curve $y^2(a+x) = x^2(a-x)$, $a > 0$. **07**

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