Seat No.: ________ Enrolment No._______

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1^{st} / 2^{nd} • EXAMINATION – SUMMER • 2014

Subject Code: 110008 **Date: 19-06-2014**

Subject Name: Mathematics - I

Time: 02:30 pm - 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q. 1. (a) (i) Given that $1 - \frac{x^2}{4} \le u(x) \le 1 + \frac{x^2}{2}$, $x \ne 0$ find $\lim_{x \to 0} u(x)$ **[2]**

(ii) Use Lagrange's Mean Value theorem to prove that **[4]**

$$\frac{b-a}{1+b^2} < \tan^{-1}b - \tan^{-1}a < \frac{b-a}{1+a^2}\ , \quad 0 < a < b$$

(b) Expand $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ in powers of x, using Maclaurin's Series. **[4]**

(c) Express cos(a + h) as a series in powers of h and hence evaluate cos 44^{o}. **[4]**

Q. 2. (a) (i) Find the equation of the tangent plane and normal line to the surface $x^3 + 2xy^2 - 7z^2 + 3y + 1 = 0$ at (1, 1, 1). **[2]**

(ii) Discuss the continuity of the function **[4]**

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2+y^2}; & (x, y) \ne (0, 0) \\ = 0; & (x, y) = (0, 0) \end{cases}$$

(b) State Euler's theorem on homogeneous function. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that **[4]**

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$

(c) If $x = r\cos\theta$ and $y = r\sin\theta$ by finding J and J' separately, show that JJ' = 1. **[4]**

Q. 3. (a) Discuss the convergence of the following series. **[6]**

(i) $\sum \frac{2+3\cos n}{n^3}$ (ii) $\sum ne^{-n^2}$ (iii) $\sum \frac{(-1)^{n-1}}{n\sqrt{n}}$

(b) Find the area outside the circle $r = 2a\cos\theta$ and inside the cardioid $r = a(1 + \cos\theta)$. **[4]**

(c) Find the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about its base. **[4]**

Q. 4. (a) (i) Evaluate $\lim\limits_{x \to a} \dfrac{\log(e^x - e^a)}{\log(x - a)}$ **[2]**

(ii) Evaluate $\iint_A y\,dx\,dy$ where A is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$ **[4]**

(b) Evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)\,dx\,dy$ by changing the order of integration. **[4]**

(c) Evaluate $\int_0^1 \int_0^1 dx\,dy$ by changing to polar co-ordinates. **[4]**

Q. 5. (a) (i) For $f(x) = x^2$, $x \in [1, 5]$, find U(f, P) and L(f, P) for P = {1, 2, 3, 4, 5} **[2]**

(ii) Find the area common to the circles $r = a$ and $r = 2a\cos\theta$ using double integration. **[4]**

(b) Find the volume bounded by the cylinder $x^2 + z^2 = 1$, $y = 0$ and $y + z = 3$ using triple integration. **[4]**

(c) Evaluate $\iiint z(x^2 + y^2)\,dv$ over the volume of the cylinder $x^2 + y^2 = 1$ intercepted by the planes z = 2 and z = 3. **[4]**

Q. 6. (a) (i) If $\overline{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$, then show that $\overline{F}$ is a solenoidal. **[2]**

(ii) Find the directional derivative of $e^{2x - y + z}$ at the point (1, 1, –1) in a direction towards the point (–3, 5, 6) **[4]**

(b) Show that $\overline{F} = y^2z^3 i + 2xyz^3 j + 3xy^2z^2 k$ is a conservative vector field and find the corresponding potential function. **[4]**

(c) Verify that $\nabla \times (\nabla \times \overline{F}) = \nabla(\nabla \cdot \overline{F}) - \nabla^2 \overline{F}$ of the vector field, $\overline{F} = 3xz^2 i - yzj + (x + 2z)k$ **[4]**

Q. 7. (a) (i) Check the convergence of the integral $\int_1^\infty \dfrac{\sin^2 x}{x^2} dx$ **[2]**

(ii) Verify Green's theorem for $\oint_C (3x - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of triangle with vertices (0, 0), (1, 0) and (0, 1). **[4]**

(b) Verify Stokes theorem for $\overline{F} = (2x - y)i - yz^2 j - y^2 zk$, where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **[4]**

(c) Find the extreme value of $x^2 + y^2 + z^2$ under the constraint $ax + by + cz = C$ **[4]**