Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–I &II (OLD) EXAMINATION – SUMMER-2019

Subject Code: 110008   **Date: 06/06/2019**

Subject Name: Maths - I

Time: 10:30 AM TO 01:30 PM   **Total Marks: 70**

Instructions:
1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 **(a)** (i) Evaluate : $\lim_{x \to 1}(1 - x)\tan\left(\frac{\pi x}{2}\right)$ **02**

(ii) Evaluate : $\lim_{x \to 0}\left(\frac{1}{x}\right)^{1-\cos x}$ **02**

(iii) Find Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$ for functions $u = x\sin y, v = y\sin x$.   **03**

(b) (i) Sketch the region and find the area bounded by ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$.   **03**

(ii) Using Lagrange's mean value theorem, prove that

$\frac{b-a}{1+b^2} < tan^{-1}b - tan^{-1}a < \frac{b-a}{1+a^2}$ **04**

Q.2 **(a)** (i) Verify Rolle's theorem for f(x) = x(x + 3)$e^{-\frac{x}{2}}$ in$-3 \le x \le 0$.   **04**

(ii) Find two non-negative numbers whose sum is 9 such that the product of one number and the square of the other is maximum.   **03**

(b) (i) Prove that $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots\right)$.   **04**

(ii) Find the absolute maximum and minimum values of $f(x) = \frac{x^3}{x+2}$ in interval $[-1, 1]$.   **03**

Q.3 **(a)** (i) Test the convergence of the series:$\sum_{n=1}^{\infty}\frac{n^n x^n}{(n+1)^n}$, $x > 0$.   **04**

(ii) Test the convergence of the series: $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^2+1}$ **03**

(b) (i) Test the convergence of the series: $\frac{1}{2} - \frac{2}{5} + \frac{3}{10} - \frac{4}{17} + \cdots$ **04**

(ii) Test the convergence of the series: $\sum_{n=1}^{\infty}\frac{n!}{n^n}$ **03**

Q.4 **(a)** (i) Find extreme values of f(x, y) = x³ + 3xy² − 3x² − 3y² + 7   **04**

(ii) Find all first and first and second order partial derivatives for f(x, y) = x²sin y + y² cos x. Hence, verify mixed derivative Theorem.   **03**

(b) (i) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$; show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2sinucos3u$.   **04**

(ii) If $u = f(x - y, y - z, z - x)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.   **03**

Q.5 **(a)** Sketch the region of integration and evaluate by reversing the order of Integration for integral $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\, dx$ **07**

(b) (i) Evaluate the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\, dy$ by changing into polar coordinates.   **04**

(ii) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{x+y} e^z dx\, dy\, dz$.   **03**

Q.6 **(a)** (i) Find directional derivative of $\emptyset = xy^2 + yz^2$ at point $(2,-1,1)$ in the direction of vector $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$.   **04**

(ii) If $\bar{r} = t^3\hat{\imath} + (2t^3 - \frac{1}{5t^2})\hat{\jmath}$, then show that $\bar{r} \times \frac{d\bar{r}}{dt} = \hat{k}$.   **03**

(b) Evaluate $\int_C \bar{F} \cdot d\bar{r}$, where, $\bar{F} = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$. Where, C is the rectangle in XY-plane bounded by $y = 0, x = a, y = b, x = 0$.   **07**

Q.7 **(a)** Verify Green's theorem for $\oint_C [(x^2 - 2xy)dx + (x^2y + 3)dy]$. Where, C is the boundary of the region bounded by $y = x^2$ and the line $y = x$.   **07**

(b) (i) Show that   **04**
$\bar{F} = (y^2 - z^2 + 3xy - 2x)\hat{\imath} + (3xz + 2xy)\hat{\jmath} + (3xy - 2xz + 2z)\hat{k}$ is both solenoidal and irrotational.

(ii) Evaluate $\oint_C [e^x dx + 2y\, dy - dz]$ by Stoke's theorem. where, C is the curve $x^2 + y^2 = 4, z = 2$.   **03**

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