

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER-1st / 2nd EXAMINATION- WINTER 2015

Subject Code: 110008

Date: 28/12/2015

Subject Name: Mathematics – 1

Time: 10:30am to 01:30pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks

Q.1 (a) Using L'hospital rule, evaluate following limits:

1. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^2}$ **03**

2. $\lim_{x \rightarrow 1} (1-x) \tan\left(\frac{\pi x}{2}\right)$ **02**

3. $\lim_{x \rightarrow 0} x^{n-1} \ln x; n > 1$ **03**

(b) Test the convergence or divergence of following series:

1. $\sum \frac{1}{n!}$ **03**

2. $\sum_{n=1}^{\infty} \frac{1}{\left(1 + \frac{1}{n}\right)^{n^2}}$ **03**

Q.2 (a) If $u = x^2y + y^2z + z^2x$, then prove that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = 3(u_{xx} + u_{yy} + u_{zz})$ **07**

(b) State Euler's theorem for homogeneous function and verify it for $u = \sqrt{x} + \sqrt{y} + \sqrt{z}$ by direct differentiation. **07**

Q.3 (a) Evaluate $\int_0^a \int_{\sqrt{ax}}^a \frac{y^2 dy dx}{\sqrt{y^4 - a^2 x^2}}$ by changing the order to integration. **07**

(b) 1. Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$ **05**

2. Using double integration find area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **02**

Q.4 (a) Verify Divergence theorem for $\vec{F} = (x^2 - yz)\mathbf{i} + (y^2 - zx)\mathbf{j} + (z^2 - xy)\mathbf{k}$ taken over the rectangular parallelepiped $0 < x < a; 0 < y < b; 0 < z < c$ **06**

(b) Do as directed

1. Evaluate $\int_C (x+y)dx - x^2 dy + (y+z)dz$ where C is $x^2 = 4y, z = x, 0 \leq x \leq 2$ **04**

2. Find the directional derivative of $f(x, y, z) = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$ **04**

- Q.5** (a) The pressure P at any point (x, y, z) in the space $P = 400xyz^2$. Find the highest pressure at the surface of a unit sphere $x^2 + y^2 + z^2 = 1$ **05**
- (b) Find the equation of tangent plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point $(2, 2, 1)$ **05**
- (c) Find out linearization of $f(x, y, z) = x^2 + 2y^2 + 3z^2 + 6$ at $(1, 1, 1)$ **04**
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- Q.6** (a) If $a < b$ then prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ **05**
- (b) Using Taylor's expansion theorem, expand $f(x) = e^x$ at $x = 0$ **05**
- (c) Prove that $\int_1^\infty \frac{\cos x}{x^2} dx$ converges. **04**
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- Q.7** (a) Expand $e^x \cos y$ at $\left(1, \frac{\pi}{4}\right)$ **05**
- (b) Test convergence of following series $\sum_{n=1}^\infty \left(\frac{n+1}{n+2}\right)^n x^n; x > 0$ **05**
- (c) Find the point of inflexion on the curve $y = 4(x+3)^3$ **04**
