

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-I & II (NEW) EXAMINATION – SUMMER-2019

Subject Code: 3110014

Date: 06/06/2019

Subject Name: Mathematics – I

Time: 10:30 AM TO 01:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- | | Marks |
|---|-------|
| Q.1 (a) Use L'Hospital's rule to find the limit of $\lim_{x \rightarrow 1} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$. | 03 |
| (b) Define Gamma function and evaluate $\int_0^\infty e^{x^2} dx$. | 04 |
| (c) Evaluate $\int_0^3 \int_{\sqrt{x}}^1 e^{y^2} dy dx$. | 07 |
| Q.2 (a) Define the convergence of a sequence (a_n) and verify whether the sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1} \right)^n$ converges or not. | 03 |
| (b) Sketch the region of integration and evaluate the integral $\iint_R (y - 2x^2) dA$ where R is the region inside the square $ x + y = 1$. | 04 |
| (c) (i) Find the sum of the series $\sum_{n \geq 2} \frac{1}{4^n}$ and $\sum_{n \geq 1} \frac{4}{(4n-3)(4n+1)}$.
(ii) Use Taylor's series to estimate $\sin 38^\circ$. | 07 |
| OR | |
| (c) Evaluate the integrals $\int_0^\infty \int_0^\infty \frac{1}{(1+x^2+y^2)^2} dx dy$ and $\int_0^1 \int_{\sqrt{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin \pi y^2}{y^2} dx dy dz$. | 07 |
| Q.3 (a) If an electrostatic field E acts on a liquid or a gaseous polar dielectric, the net dipole moment P per unit volume is $P(E) = \frac{e^{E+e^{-E}}}{e^E - e^{-E}} - \frac{1}{E}$. Show that $\lim_{E \rightarrow 0^+} P(E) = 0$. | 03 |
| (b) For what values of the constant k does the second derivative test guarantee that $f(x, y) = x^2 + kxy + y^2$ will have a saddle point at $(0,0)$? A local minimum at $(0,0)$? | 04 |
| (c) Find the series radius and interval of convergence for $\sum_{n=0}^\infty \frac{(3x-2)^n}{n}$. For what values of x does the series converge absolutely? | 07 |
| OR | |
| Q.3 (a) Determine whether the integral $\int_0^3 \frac{dx}{x-1}$ converges or diverges. | 03 |
| (b) Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1, x = 4$ about the line $y = 1$. | 04 |

(c) Check the convergence of the series 07

$$\sum_{n=1}^{\infty} \frac{(\ln n)^3}{n^3} \text{ and } \sum_{n=0}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n}).$$

Q.4 (a) Show that the function $f(x, y) = \frac{2x^2y}{x^4+y^2}$ has no limit as (x, y) 03

approaches to $(0, 0)$.

(b) Suppose f is a differentiable function of x and y and $g(u, v) = f(e^u + \sin v, e^u + \cos v)$. Use the following table to calculate $g_u(0, 0), g_v(0, 0), g_u(1, 2)$ and $g_v(1, 2)$. 04

	f	g	f_x	f_y
$(0, 0)$	3	6	4	8
$(1, 2)$	6	3	2	5

(c) Find the Fourier series of 2π -periodic function $f(x) = x^2, 0 < x < 2\pi$ and hence deduce that $\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$. 07

OR

Q.4 (a) Verify that the function $u = e^{-\alpha^2 k^2 t} \cdot \sin kx$ is a solution of the heat conduction equation $u_t = \alpha^2 u_{xx}$. 03

(b) Find the half-range cosine series of the function 04

$$f(x) = \begin{cases} 2, & -2 < x < 0 \\ 0, & 0 < x < 2 \end{cases}.$$

(c) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$. 07

Q.5 (a) Find the directional derivative $D_u f(x, y)$ if $f(x, y) = x^3 - 3xy + 4y^2$ and u is the unit vector given by angle $\theta = \frac{\pi}{6}$. What is $D_u f(1, 2)$? 03

(b) Find the area of the region bounded by the curves $y = \sin x, y = \cos x$ and the lines $x = 0$ and $x = \frac{\pi}{4}$. 04

(c) Prove that $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ is diagonalizable and use it to find A^{13} . 07

OR

Q.5 (a) Define the rank of a matrix and find the rank of the matrix $A = \begin{bmatrix} 2 & -1 & 0 \\ 4 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$. 03

(b) Use Gauss-Jordan algorithm to solve the system of linear equations $2x_1 + 2x_2 - x_3 + x_5 = 0$ 04

$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$

$$x_1 + x_2 - 2x_3 - x_5 = 0$$

$$x_3 + x_4 + x_5 = 0$$

(c) State Cayley-Hamilton theorem and verify it for the matrix 07

$$A = \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}.$$