

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-I & II (OLD) EXAMINATION – SUMMER-2019

Subject Code: 110009

Date: 01/06/2019

Subject Name: Maths - II

Time: 10:30 AM TO 01:30 PM

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- $2x + y + 2z = 0$
- Q-1 (a)** (i) Find the value of λ so that the equations $x + y + 3z = 0$; have a non trivial solution. 4
- $4x + 3y + \lambda z = 0$
- (ii) Verify Cauchy-Schwarz inequality for the vectors $\vec{u} = (-3, 1, 0)$, $\vec{v} = (2, -1, 3)$. 3
- (b)** Solve the following equations by Gauss elimination and back substitution, 7
- $x + y + 2z = 9$
 $2x + 4y - 3z = 1$
 $3x + 6y - 5z = 0$
- Q-2 (a)** (i) Obtain the reduced row echelon form of the matrix $\begin{bmatrix} 1 & 3 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 2 & 4 & 3 & 4 \\ 3 & 7 & 4 & 8 \end{bmatrix}$. 4
- (ii) Find the rank of the matrix, if $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$. 3
- (b)** Use row operation to find A^{-1} , if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. 7
- Q-3 (a)** (i) Find the eigen values and eigen vectors of the matrix, $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. 4
- (ii) Using Cayley-Hamilton theorem, find A^2 , if $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. 3
- (b)** Find a matrix P that diagonalizes $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ and hence find A^{10} . 7
- Q-4 (a)** (i) Reduce $S = \{(1, 0, 0), (0, 1, -1), (0, 4, -3), (0, 2, 0)\}$ to obtain a basis for R^3 . 4
- (ii) Determine whether or not the vectors $\{(1, 2, 2), (2, 1, 2), (2, 2, 1)\}$ in R^3 are linearly independent. 3
- (b)** Let $V = \{(x, y) | x, y \in R, y > 0\}$, let $(a, b), (c, d) \in V, \alpha \in R$.
 Define $(a, b) + (c, d) = (a + c, b \cdot d)$ and $\alpha \cdot (a, b) = (\alpha a, b^\alpha)$. Prove that V is a vector space. 7
- Q-5 (a)** (i) Show that the transformation $T : R^2 \rightarrow R^2$, where $T(x, y) = (2x - y, x - y)$ is a linear transformation. 4
- (ii) Express the quadratic form $Q(x, y) = 2x^2 + 3y^2 + 6xy$, in matrix notation. 3

- (b) Find the rank and nullity of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ 7
- Q-6** (a) (i) Find a basis for the orthogonal complement of the subset of R^3 spanned by the vectors $\bar{v}_1 = (1, -1, 3)$, $\bar{v}_2 = (5, -4, -4)$, $\bar{v}_3 = (7, -6, 2)$. 4
- (ii) Determine whether the linear transformation $T : R^2 \rightarrow R^3$, where $T(x, y) = (x, y, x + y)$ is one-one. 3
- (b) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $S = \{(1, 1, 1), (-1, 1, 0), (1, 2, 1)\}$ into an Orthonormal basis. 7
- Q-7** (a) (i) Prove that $A = \begin{bmatrix} 1 & 3+4i & -2i \\ 3-4i & 2 & 9-7i \\ 2i & 9+7i & 3 \end{bmatrix}$ is a Hermitian matrix. 4
- (ii) Let R^2 have the Euclidean inner product. Find the cosine of the angle θ between the vectors $\bar{u} = (4, 3, 1, -2)$ and $\bar{v} = (-2, 1, 2, 3)$. 3
- (b) Find the least square solution of the linear system $AX = b$ and find the orthogonal projection of b onto the column space of A , where $A = \begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ 7
