Seat No.: ________ *Enrolment No.*___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 110009 **Date: 05-01-2015**

Subject Name: Mathematics - II

Time: 10:30 am - 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find the Rank of the matrix $\begin{bmatrix} 0 & -1 & 2 & 3 \\ 2 & 3 & 4 & 5 \\ 1 & 3 & -1 & 2 \\ 3 & 2 & 4 & 1 \end{bmatrix}$ **05**

(b) Solve the following system of equations using Gauss Elimination method **05**

$$3x + 3y + 2z = 1 \;, \quad x + 2y = 4$$
$$10y + 3z = -2 \;, \quad 2x - 3y - z = 5$$

(c) Find k , l and m so that $\begin{bmatrix} -1 & k & -i \\ 3-5i & 0 & m \\ l & 2+4i & 2 \end{bmatrix}$ is Hermitian. **04**

Q.2 **(a)** Show that the set of all pairs of real numbers of the form $(1, x)$ with the operations defined as $(1, x) + (1, y) = (1, x + y)$ and $k(1, x) = (1, kx)$ is a vector space. **05**

(b) Express the vector $(6, 11, 6)$ as a linear combination of $(2, 1, 4)$, $(1, -1, 3)$, $(3, 2, 5)$ **05**

(c) Find the condition on a , b , c so that the vector $v = (a, b, c)$ is in the span of $\{v_1, v_2, v_3\}$ where $v_1 = (2, 1, 0)$, $v_2 = (1, -1, 2)$, $v_3 = (0, 3, -4)$ **04**

Q.3 **(a)** Check whether the set $\{2 + x + x^2,\ x + 2x^2, 4 + x\}$ of polynomials is linearly dependent or independent in P_2 **05**

(b) Find a basis for the subspace of P_2 spanned by the vectors $1 + x, x^2, -2 + 2x^2, -3x$ **05**

(c) Find a basis for the row and column subspaces of $\begin{bmatrix} 1 & 4 & 5 & 4 \\ 2 & 9 & 8 & 2 \\ 2 & 9 & 9 & 7 \\ -1 & -4 & -5 & -4 \end{bmatrix}$ **04**

Q.4 **(a)** Show that $T : R^3 \to R^2$ defined by $T(x, y, z) = (2x - y + z,\ y - 4z)$ is a linear transformation. **05**

(b) Consider the basis $S = \{v_1, v_2\}$ for R^2 where $v_1 = (-2, 1)$ and $v_2 = (1, 3)$. **05**

Let $T : R^2 \to R^3$ be the linear transformation such that $T(v_1) = (-1, 2, 0)$ and $T(v_2) = (0, -3, 5)$ then find the formula of $T(x, y)$

(c) Let $T : R^2 \to R^2$ be the linear transformation defined by $T(x, y) = (2x - y,\ -8x + 4y)$ then find a basis for kernel of T and range of T **04**

Q.5 **(a)** Let $T : R^3 \to R^3$ be the linear transformation defined by $T(x, y, z) = (x + 2y + z,\ 2x - y,\ 2y + z)$ then find the matrix of T with respect to the basis $\{(1, 0, 1), (0, 1, 1), (0, 0, 1)\}$ **05**

(b) Let $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$ then check whether $< u, v > = u_1v_1 - u_2v_2 + u_3v_3$ defines an inner product on R^3 **05**

(c) For $p = a_0 + a_1x + a_2x^2$ and $q = b_0 + b_1x + b_2x^2$ let the inner product on P_2 be defined as $< p, q > = a_0b_0 + a_1b_1 + a_2b_2$. Let $p = 3 - x + x^2$ and $q = 2 + 5x^2$ then find $\|p\|$, $\|q\|$ and $d(p, q)$ **04**

Q.6 **(a)** For $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}$ and $B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}$ let the inner product on M_{22} be defined as $< A, B > = a_1a_2 + b_1b_2 + c_1c_2 + d_1d_2$. Let $A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$ then verify Cauchy-Schwarz inequality and find the angle between A and B **05**

(b) Show that the set of vectors $v_1 = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$, $v_2 = \left(-\frac{1}{2}, \frac{1}{2}, 0\right)$ and $v_3 = \left(\frac{1}{3}, \frac{1}{3}, -\frac{2}{3}\right)$ is orthogonal in R^3 and then convert it into an orthonormal set **05**

(c) Find the algebraic and geometric multiplicity of each of the eigen value of **04**

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

Q.7 **(a)** Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1} **05**

(b) Find a non singular matrix which diagonalizes $\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ **05**

(c) Find the maximum and minimum values of the quadratic form $x^2 + y^2 + 4xy$ subject to the constraint $x^2 + y^2 = 1$ and also determine the values of x and y at which the maximum and minimum occur **04**

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