Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 1st / 2nd • EXAMINATION – SUMMER • 2014

Subject Code: 110015 **Date: 16-06-2014**

Subject Name: Vector Calculus and Linear Algebra

Time: 02:30 pm - 05:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

1 If a force $\bar{F} = 2x^2\, yi + 3xyj$ displace a particle in the xy-plane from (0,0) to (1,4) along a curve $y = 4x^2$. Find work done. **14**

2 For what values of constant k does the system $x - y = 3, 2 - 2y = k$ Have no solution ? Exactly one solution ? Infinitely many solution ?

3 Solve following equations by Cramer's rule.

$x_1 + x_2 + x_3 = 9$

$2x_1 + 5x_2 + 7x_3 = 52$

$2x_1 + x_2 - x_3 = 0$

4 $T: R^2 \to R^2, T(x,y) = (2x + y, x - 2y)$.

$$Is\ T\ one - one?\ If\ so\ find\ T^{-1}$$

5 Find the co-ordinates of a polynomial $p = 5 + 11x + 2x^2$ relative to the basis S={ $1 - x, 1 + x, 1 - x^2$ }$of\ P_2(R)$.

6 Determine the algebraic multiplicity of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$.

7 Let W= span{ (0,1,0) , (-4/5,0,3/5) }
Express W= (1,1,1) in the form W= $w_1 + w_2$ where $w_1 \in$ w , $w_2 \in w^{\perp}$

Q.2 (a) (1) Verify whether the following matrices are Hermitian or skew Hermitian or neither. Give reason. **02**

(i) $\begin{bmatrix} a & c + id \\ c - id & b \end{bmatrix}$ (ii) $\begin{bmatrix} 2i & 1 + i & -3 + 2i \\ -1 + i & 0 & 2 - i \\ 3 + 2i & -2 - i & -3i \end{bmatrix}$.

(2) Solve the following set of equations by Gauss Jordan method. **05**

$$x + y + 2z = 8$$
$$-x - 2y + 3z = 1$$
$$3x - 7y + 4z = 10$$

(b) (1) Find the inverse of matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ using row operations. **04**

(2) Reduce the following matrix to reduced row echelon form. **03**

$$\begin{pmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

Q.3 (a) (1) Show that $< \bar{u}, \bar{v} > = 9u_1v_1 + 4u_2v_2$ is an inner product on R^2 generated by A= $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$. **02**

(2) Using Gramm-schmidt process construct an orthonormal basis for R^3 Whose basis is the set { (2,1,3) ,(1,2,3) ,(1,1,1) } **05**

(b) (1) Find an angle between t and sint for V is an inner product space of all continuous functions on$[0,\pi]$ $with$ the inner product defined by **04**

$< \bar{f}, \bar{g} >= \int_0^{\pi} \bar{f}(t)\, \bar{g}(t)\, dt$.

(2) Show that $< \bar{x}, \bar{y} > = x_1y_1 - x_1y_2 - x_2y_1 + 2x_2y_2$ on R^2 is an inner product. **03**

Q.4 (a) (1) (i)) Check whether V = R^2 is a vector space defined by the operations **04**

$(u_1, u_2) \oplus (v_1, v_2)\ ) = (u_1 + 2v_1, u_2 + 2v_2)$

$\propto \odot (u_1, u_2) = (\alpha u_1 - 1, \alpha u_2 + 2)$

(ii)S $= \{ (x_1, x_2, x_3) / x_1 + 2x_2 = 1\}$ Check whether S is a subspace of R^3.

(2) Express $p(x) = 6 + 11x + 6x^2$ as a linear combination of the following. **03**

$p_1 = 2 + x + 4x^2\ , p_2 = 1 - x + 3x^2\ , p_3 = 3 + 2x + 5x^2$.

(b) (1) Verify dimenesion theorem of the matrix **04**

$$A = \begin{pmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{pmatrix}$$

(2) Show that the following set of vectors is a basis for $M_{22.}$ 03 **03**

$$S = \left\{ \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \right\}$$

Q.5 (a) Find eigen values and eigen vectors of the matrix **04**

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

(b) Verify Caley-Hamilton theorem for the matrix **05**

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1}

(c) Find the canonical form of the quadratic form **05**

$Q = 2x_1{}^2 + 3x_2{}^2 + 2x_3{}^2 + 2x_1x_3$ using orthogonal transformation.

Find index ,rank ,signature of the quadratic form

Q.6 (a) Find a matrix for the linear transformation L: $p_3 \to M_{22}$ defined by **04**

L$(ax^3 + bx^2 + cx + d) = \begin{bmatrix} -3a - 2c & -b + 4d \\ 4b - c + 3d & -6a - b + 2d \end{bmatrix}$ with respect to the standard basis B$(x^3, x^2, x, 1)$ *and*

$C = \{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \}$

(b) Find a formula for $T(x_1, x_2)$ and use it to find $T(5, -3)$ for the basis S=$\{v_1, v_2\}$ for R^2 where $v_1 = (1,1), v_2 = (1,0)$. Let T: $R^2 \to R^2$ be a linear transformation such that $T(v_1)$ =(1, , −2), $T(v_2)$=(−4,1). **05**

(c) If T: $R^3 \to R^3$ is a linear transformation given by **05**

T(x,y,z) = (x+y−z,x−2y+z, −2x−2y+2z).Find the basis of ker(T) and R(T).

Q.7 (a) (1) Find the derivative of f(x,y)=$xe^y + \cos(xy)$ at the point (2,0) in the direction of A=$3i - 4j$. **04**

(2) Find constants a,b,c so that

$V = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)$ is irrotational.

(b) State Green's theorem and also evaluate the integral **05**

$\oint (6y + x)dx + (y + 2x)dy$ where C: the circle

$(x - 2)^2 + (y - 3)^2 = 4$.

(c) Find the flux of $F = yzj + z^2$k outward through the surface S cut from the cylinder $y^2 + z^2 = 1, z \geq 0$ by the planes x=0 and x=1. **05**

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