

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 110015

Date: 05-01-2015

Subject Name: Vector Calculus and Linear Algebra

Time: 10:30 am - 01:00 pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (1) Solve the following system of equations (if possible) using Gauss elimination method. $2x + y - z = 4$, $x - y + 2z = -2$, $-x + 2y - z = 2$ (14)

(2) If for two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$. Find angle between $\vec{a}$ and $\vec{b}$.

(3) Show that the linear operator $T : R^2 \rightarrow R^2$ defined by the equation $w_1 = 2x + y$, $w_2 = 3x + 4y$ is one-one and find $T^{-1}(w_1, w_2)$.

(4) Prove or disprove that the set $W = \{(a, 2a, a + 1) | a \in R\}$ is or not vector-subspace of R^3 .

(5) Find two vectors in R^2 with Euclidian Norm whose inner product with $(-3, 1)$ is zero.

(6) Find the eigenvalue and eigenvector of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

(7) Let $S = \{v_1, v_2, v_3\}$ be the basis for R^3 in the $v_1(1, 2, 1)$, $v_2(2, 9, 0)$, $v_3(3, 3, 4)$ find the coordinate vector of $V = (5, -1, 9)$.

Q.2 (a) (1) Find A^{-1} by Gauss Jordan method where $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$. (3)

(2) Solve following equation by Cramer's rule. $x + 2z = 6$, $-3x + 4y + 6z = 30$, $-x - 2y + 3z = 8$. (4)

(b) (1) Investigate for what values of λ and μ the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (i) no solution (ii) a unique solution (iii) an infinite number of solution. (4)

(2) Reduce the following matrix into reduced row echelon form (3)

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

Q.3 (1) Determine which sets are vector space or not under the given operation. The set all triple of real no $\{x, y, z\}$ with the operations (5)

$$(x, y, z) + (x', y', z') = (x + x', y + y', z + z') \text{ and } (x, y, z) = (kx, y, z)$$

(2) Define basis and Determine the dimension and basis for the solution space of

$$\text{the system } 3x + y + z + w = 0 \quad 5x - y + z - w = 0 \quad (4)$$

(3) State Dimension theorem and verify that for the given matrix

$$A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix} \quad (5)$$

Q.4 (1) Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3 , $v_1(1,2,1)$ $v_2(2,9,0)$ $v_3(3,3,4)$ and let

$T : R^3 \rightarrow R^2$ be the linear transformation such that $T(v_1) = (1,0)$

$T(v_2) = (-1,1)$, $T(v_3) = (0,1)$. Find a formula for $T(x_1, x_2, x_3)$ and use the form formula to find $T(7,13, 7)$. (5)

(2) Determinew whether the given vectors $v_1 = (2, -1, 3)$ $v_2 = (4, 1, 2)$

$v_3 = (8, -1, 8)$ span R^3 . (3)

(3) Let $v_1 = (1, -1, 0)$ $v_2 = (0, 1, -1)$ $v_3 = (0, 0, 1)$ be elements in R^3 . Verify the set of vectors (v_1, v_2, v_3) is linearly independent or linearly dependent. (3)

(4) Find the basis for the row space for the matrix A

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix} \quad (3)$$

Q.5 (1) Find the eigenvalue and eigenvector for the matrix (3)

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

(2) find a matrix P that diagonalizes A and determine $P^{-1}AP$ (5)

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

(3) Verify Caley-Hamilton theorem for the matrix. (3)

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

(4) Reduce the quadratic form $Q(x, y, z) = 3x^2 + 3z^2 + 4xy + 8xz + 8yz$ into canonical form using linear transformaion. (3)

Q.6 (1) Let V be an inner product space and let $u, v \in V$ Prove that

$$\langle u, v \rangle = \frac{1}{4} \|u + v\|^2 - \frac{1}{4} \|u - v\|^2 \quad (2)$$

(2) Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$ be a vector in R^2 Verify that the weighted Euclidean inner product on R^2 $\langle u, v \rangle = 3u_1v_1 + 2u_2v_2$. (4)

(3) Let R^3 have the Euclidean inner Product. Use the Gram-Schmidt process to transform the bases $\{u_1, u_2, u_3\}$ into orthogonal bases $u_1 = (1,1,1)$

$u_2 = (-1,1,0) \quad u_3 = (1,2,1)$ (4)

(4) Find a least square solution of the inconsistent system $Ax=b$ for

$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}.$ (4)

Q.7 (1) If $\phi = 3x^2y - y^3z^2$, find the grad ϕ at the point $(1,-2,-1)$. (2)

(2) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at the point $(2,-1,2)$ in the Direction $(2,3,6)$. (4)

(3) Using Green's theorem, evaluate $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$ Where c is the

Boundary of the region bounded by $y^2 = x$ and $x^2 = y$. (4)

(4) Evaluate $\iint_s 6xyds$ where s is the portion of the plane $x + y + z = 1$ that lies in front of Yz plane (4)
