

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER-I & II EXAMINATION – WINTER 2015

Subject Code: 110015

Date: 21/12/2015

Subject Name: Vector Calculus and Linear Algebra

Time: 10:30am to 01:30pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)(i)** Verify the Cauchy-Schwarz inequality **04**
- (1) $u = (-2, 3, 1, -1)$, $v = (7, 1, -4, -2)$
(2) $u = (-3, 1, 0)$, $v = (2, -1, 3)$
- (ii)** Given $u = (9, 3, -4, 0, 1)$, $v = (0, -3, 2, -1, 7)$ then compute **03**
- (1) $u - 4v$ (2) $u \cdot u$ (3) $d(u, v)$
- (b)(i)** Use Cramer's rule to solve linear system **04**
- $x + 2y + z = 5, 3x - y + z = 6, x + y + 4z = 7$
- (ii)** Test the consistency of the system **03**
- $x + 2y + z = 3, 2x + y + 3z = 5, 2x + 4y + 2z = 7$
- Q.2 (a)(i)** Show that the vectors $v_1 = (2, 1, 1), v_2 = (1, 2, 2), v_3 = (1, 1, 1)$ are Linearly Independent. **04**
- (ii)** Find A^{-1} using row operations if $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 7 & 6 \\ 2 & 7 & 7 \end{bmatrix}$ **03**
- (b)(i)** Express $(2, 5, -3)$ as linear combination of $\{(2, -1, 1), (1, 1, -1), (4, 2, 1)\}$ in R^3 . **04**
- (ii)** Determine whether the following polynomials span P_2 **03**
- $p_1(x) = 1 - x + 2x^2, p_2(x) = 5 - x + 4x^2, p_3(x) = -2 - 2x + 2x^2$
- Q.3 (a)(i)** Prove that the set of all positive real numbers with operations **05**
- $x + y = xy$ and $kx = x^k$ is a real vector space.
- (ii)** For what choices of the parameters the following system have a unique solution ? **02**
- $x + 2y = a$ and $3x + 4y = b$

(b)(i) Find the rank of $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ 04

(ii) Determine if the set of vectors will be a basis for $\mathbb{R}^3$. 03

$v_1 = (1, 2, 1), v_2 = (2, 9, 0), v_3 = (3, 3, 4)$

Q.4 (a)(i) A function $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined on the basis $B = \{(1, 1, 1), (1, 1, 0), (1, 0, 0)\}$ of $\mathbb{R}^3$ by $T(1, 1, 1) = (1, 2, 3), T(1, 1, 0) = (2, 3, 4), T(1, 0, 0) = (3, 4, 5)$ then find $T(x)$ for any x . 04

(ii) Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $T(x, y) = (x + y, x - y)$ show that T is one-one also find the formula for $T^{-1}(x, y)$. 03

(b)(i) Verify Cayley-Hamilton theorem for the following matrix 04

$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

(ii) Determine a basis for the null space of the matrix 03

$\begin{bmatrix} 1 & 2 & -3 & 2 & -3 \\ 1 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 1 & 2 & 5 & -6 & -3 \end{bmatrix}$

Q.5 (a)(i) Evaluate $\int_C ydx + xdy + zdz$, where C is given by $x = \cos t, y = \sin t, z = t^2, 0 \leq t \leq 2\pi$ 04

(ii) Evaluate $\int_C F \cdot dr$ where $F = \frac{yi - xj}{(x^2 + y^2)}$ and C is the circle $x^2 + y^2 = 1$ traversed counterclockwise. 03

(b)(i) Using Green's theorem evaluate $\oint_C xydy - y^2dx$, where C is the square cut from the first quadrant by the lines $X=1$ and $Y=1$. 04

(ii) Find the value of the constant λ such that the vector field defined by $F = (2x^2y^2 + z^2)i + (3xy^3 - x^2z)j + (\lambda xy^2z + xy)k$ is solenoidal. 03

Q.6 (a)(i)

Find eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$.

04

(ii) Determine the algebraic and geometric multiplicity of $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

03

(b)(i) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then using Cayley-Hamilton theorem find A^2 and A^{-1} .

04

(ii) Find the matrix P which diagonalises $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

03

Q.7 (a)(i) Let R^4 have the Euclidian inner product then find the cosine of the angle between vectors $u = (4, 3, 1, -2), v = (-2, 1, 2, 3)$. **04**

(ii) Using Gram-Schmidt process orthonormalize the set of linearly independent vectors $u_1 = (1, 0, 1, 1), u_2 = (-1, 0, -1, 1), u_3 = (0, -1, 1, 1)$ of R^4 . with standard inner product. **03**

(b) (i) Find the least squares solution of the linear system $Ax=b$ for **04**

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}, b = \begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix}$$

(ii) Find the matrix of the quadratic form $x^2 + 2y^2 - 7z^2 - 4xy + 8xz + 5yz$. **03**
