Seat No.: ________ *Enrolment No.*___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – I-II (NEW) • EXAMINATION – WINTER • 2014

Subject Code: 2110015 **Date: 05-01-2015**

Subject Name: Vector Calculus and Linear Algebra

Time: 10:30 am - 01:30 pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1

(a) Choose the appropriate answer for the following MCQs. **(07)**

1. If $\left|\vec{a}\times\vec{b}\right| = \vec{a}\cdot\vec{b}$ then the angle between two vectors $\vec{a}$ and $\vec{b}$ is

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

2. If $\vec{a} = 2\vec{i} - 3\vec{j} + \vec{k}$ then the $\left|\vec{a}\right| =$

(a) $\sqrt{-4}$ (b) $\sqrt{4}$ (c) $\sqrt{13}$ (d) $\sqrt{14}$

3. If $\vec{F}$ is conservative then

(a) $\nabla\times\vec{F} = 0$ (b) $\nabla\times\vec{F} \neq 0$ (c) $\nabla\vec{F} = 0$ (d) $\nabla\cdot\vec{F} = 0$

4. If $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ then the determinant of A is

(a) -2 (b) 1 (c) -1 (d) 0

5. If $A = \begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix}$ then the determinant of A is

(a) 0 (b) -1 (c) 1 (d) 2

6. The characteristic equation for the matrix $A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$

(a) $(\lambda - 2)^2 = 0$ (b) $\lambda + 2 = 0$ (c) $(\lambda - 2)(\lambda + 2) = 0$ (d) $\lambda - 2 = 0$

7. If A is a matrix with 5 columns and nullity of A = 2 then rank(A) is

(a) 5 (b) 2 (c) 3 (d) 4

(b) Choose the appropriate answer for the following MCQs. **(07)**

1. If $\left|\vec{a}+\vec{b}\right| = \left|\vec{a}-\vec{b}\right|$ then the angle between two vectors $\vec{a}$ and $\vec{b}$

(a) 30^0 (b) 45^0 (c) 60^0 (d) 90^0

2. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then the divergence of $\vec{r}$ is

(a) 2 (b) -2 (c) 3 (d) -3

3. If A and kA have same rank then what can be said about k?

(a) zero (b) non-zero (c) positive (d) negative

4. If V is a vector space having a basis B with n elements then dim(V) =

(a) < n (b) > n (c) n (d) none of these

5. For a $n\times n$ matrix A, Which one of the following statements does not imply the other?

(a) A is not invertible (b) $\det(A) \neq 0$ (c) $rank(A) = n$

(d) $\lambda = 0$ is not an eigen-value of A

6. If a complex number $\lambda \neq 0$ is an eigen value of 2×2 real matrix A, then which one of the following is not true?

(a) $\bar{\lambda}$ is also an eigen-value of A (b) $\det(A) \neq 0$ (c) $rank(A) = 2$

(d) A is not invertible

7. If a 3×3 matrix A is diagonalizable then which one of the following is true?
(a) A has 2 distinct eigen-values.
(b) A has 2 linearly independent eigen-vectors.
(c) A has 3 linearly independent eigen-vectors.
(d) none of these

Q.2

(a) Show that $A=\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}$ is orthogonal. **(03)**

(b) Is T: $R^3 \rightarrow R^3$ defined by $T(x,y,z)=(x+3y,y,z+2x)$ linear ? Is it one-to-one, onto or both ? Justify. **(04)**

(c) Define rank of a matrix. Determine the rank of the matrix **(07)**

$$A=\begin{bmatrix}3 & 4 & 5 & 6 & 7\\ 4 & 5 & 6 & 7 & 8\\ 5 & 6 & 7 & 8 & 9\\ 10 & 11 & 12 & 13 & 14\\ 15 & 16 & 17 & 18 & 19\end{bmatrix}$$

Q.3

(a) Find A^{-1} for $A=\begin{bmatrix}1 & 0 & 1\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$, if exists. **(03)**

(b) Obtain the reduced row echelon form of the matrix $A=\begin{bmatrix}1 & 3 & 2 & 2\\ 1 & 2 & 1 & 3\\ 2 & 4 & 3 & 4\\ 3 & 7 & 4 & 8\end{bmatrix}$ and hence find the rank of the matrix A. **(04)**

(c) State rank-nullity theorem. Also verify it for the linear transformation T: $R^3 \rightarrow R^2$ defined by $T(x,y,z)=(x+y+z,x+y)$. **(07)**

Q.4

(a) If $A=\begin{bmatrix}1 & 2 & -3\\ 0 & 2 & 3\\ 0 & 0 & 2\end{bmatrix}$ then find the eigen values of A^T and $5A$. **(03)**

(b) Solve the system of linear equations by Cramer's Rule: **(04)**

$$\begin{aligned} x+2y+z&=5\\ 3x-y+z&=6\\ x+y+4z&=7\end{aligned}$$

(c) Verify Green's Theorem in the plane for $\oint_C (3x^2-8y^2)dx+(4y-6xy)dy$, where C is the boundary of the region defined by $y^2=x$ and $x^2=y$ **(07)**

Q.5

(a) Find $grad(\phi)$, if $\phi=\log(x^2+y^2+z^2)$ at the point (1, 0, -2). **(03)**

(b) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at the point (2, -1, 2) **(04)**

(c) (1) Let T: $R^2 \rightarrow R^3$ be the linear transformation defined by $T(x,y)=(y,-5x+13y,-7x+16y)$. Find the matrix for the transformation T with respect to the basis $B=\{(3,1)^T,(5,2)^T\}$ for R^2 and $B'=\{(1,0,-1)^T,(-1,2,2)^T,(0,1,2)^T\}$ for R^3. **(05)**

(2) Find a basis for the orthogonal complement of the subspace W of R^3 defined as W = {(x,y,z) in R^3| -2x + 5y – z = 0} **(02)**

Q.6 **(a)** Show that $\vec{F}=(y^2-z^2+3yz-2x)\vec{i}+(3xz+2xy)\vec{j}+(3xy-2xz+2z)\vec{k}$ is both solenoidal and irrotational. **(03)**

(b) A vector field is given by $\vec{F}=(x^2+xy^2)\vec{i}+(y^2+x^2y)\vec{j}$. Find the scalar potential. **(04)**

(c) (1) Show that the set of all pairs of real numbers of the form $(1,x)$ with the operations defined as $(1,x_1)+(1,x_2)=(1,x_1+x_2)$ and $k(1,x)=(1,kx)$ **(05)**

(2) Verify Caylay-Hamilton Theorem for the matrix $A=\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & 1\\ 0 & 1 & -1\end{bmatrix}$ **(02)**

Q.7 **(a)** Find a basis for the subspace of P_2 spanned by the vectors $1+x,\ x^2,\ -2+2x^2,\ -3x$ **(03)**

(b) Let R^3 have the Euclidean inner product. Transform the basis $S=\{(1,0,0),(3,7,-2),(0,4,1)\}$ into an orthonormal basis using the Gram-Schmidt ortho-normalization process. **(04)**

(c) Evaluate $\iint_S \vec{F}\cdot\bar{n}dS$ where $\vec{F}=yz\vec{i}+xz\vec{j}+xy\vec{k}$ and S is the surface of the sphere $x^2+y^2+z^2=1$ in the first octant. **(07)**

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