Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–I & II (NEW) EXAMINATION – WINTER 2015

Subject Code: 2110015 **Date:21/12/2015**

Subject Name: Vector Calculus and Linear Algebra

Time: 10:30am to 01:30pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Mark

Q.1 **Objective Question (MCQ)**

(a) **07**

1. Eigen values of [redacted] & [redacted] are same if matrix [redacted] is
(a) Symmetric (b) Orthogonal (c) skew symmetric (d) None of these

2. Rank of [redacted] invertible matrix is
(a)1 (b)2 (c)3 (d)4

3. [redacted] is solenoidal vector, If [redacted] is
(a) [redacted] (b) 1 (c) 0 (d) -1

4. Let [redacted] be a hermition matrix, then [redacted] is
(a) [redacted] (b) [redacted] (c) [redacted] (d) [redacted]

5. If [redacted], then eigen values of [redacted] are
(a) 1,-1 (b) 0,2 (c) 1,1 (d) 0,8

6. Which set from $S_1 = \{a_0 + a_1x + a_2x^2 / a_0 = 0\}$ and $S_2 = \{a_0 + a_1x + a_2x^2 / a_0 \neq 0\}$ is subspace of P_2?
(a) [redacted] (b) [redacted] (c) [redacted] (d) none of these

7. For which value of k vectors u= (2, 1, 3) and v= (1, 7, k) are orthogonal?
(a) -3 (b) -1 (c) 0 (d) 2

(b) **07**

1. Let [redacted] be one to one linear transformation then the dimension of ker(T) is
(a)0 (b) 1 (c) 2 (d)3

2. The column vector of an orthogonal matrix are
(a) orthogonal (b) orthonormal (c) dependent (d) none of these

3. If **r** = xi+yj+zk then div (**r**) is
(a) **r** (b) 0 (c) 1 (d) 3

4. The number of solution of the system of equation AX=0 (where A is a singular matrix) is
(a) 0 (b) 1 (c) 2 (d) infinite

5. If the value of line integral does not depend on path C then **F** is
(a) solenoidal (b) incompressible (c) irrotational (d) none of these

6. A Cayley-Hamilton theorem hold for _________ matrices only
(a) singular (b) all square (c) null (d) a few rectangular

7. If [redacted], then rank of matrix A is
(a) 1 (b) 0 (c) 2 (d) 4

Q.2 **(a)** Determine whether the vector field $\mathbf{u} = y^2\hat{i} + 2xy\hat{j} - z^2\hat{k}$ is solenoidal at a point (1,2,1). **03**

(b) Prove that the matrix $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$ is a Hermition and iA is a skew Hermition matrix. **04**

(c) For which value of λ and k the following system have (i) no solution (ii) unique solution (iii) an infinite no. of solution. **07**

$$\begin{aligned} x + y + z &= 6 \\ x + 2y + 3z &= 10 \\ x + 2y + \lambda z &= k \end{aligned}$$

Q.3 **(a)** Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$ **03**

(b) Find the inverse of matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ by Gauss-Jordan method **04**

(c) Consider the basis S = { v_1, v_2 } for R^2 where v_1 = (-2,1), v_2 = (1,3) and let $T: R^2 \to R^3$ be the linear transformation such that T(v_1) = (-1,2,0), T(v_2) = (0,-3,5). Find a formula for T(x_1,x_2) and use the formula to find T(2,-3). **07**

Q.4 **(a)** Express $p(x) = 7 + 8x + 9x^2$ as linear combination of **03**

$p_1 = 2 + x + 4x^2$, $p_2 = 1 - x + 3x^2$, $p_3 = 2 + x + 5x^2$.

(b) Solve the system by Gaussian elimination method **04**

$x + y + z = 6 \quad x + 2y + 3z = 14 \quad 2x + 4y + 7z = 30$

(c) Let R^3 have standard Euclidean inner product. Transform the basis S = { v_1, v_2, v_3} into an orthonormal basis using Gram-Schmidt Process where v_1 = (1,1,1), v_2 = (-1,1,0), v_3 = (1,2,1). **07**

Q.5 **(a)** Find the nullity of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ **03**

(b) Find the least square solution of the linear system Ax = b and find the orthogonal projection of b onto the column space of A where **04**

$A = \begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

(c) Show that $S=\left\{\begin{bmatrix}1 & 2\\ 1 & -2\end{bmatrix},\begin{bmatrix}0 & -1\\ -1 & 0\end{bmatrix},\begin{bmatrix}0 & 2\\ 3 & 1\end{bmatrix},\begin{bmatrix}0 & 0\\ -1 & 2\end{bmatrix}\right\}$ is a basis for M_{22}. **07**

Q.6 (a) Verify Pythagorean theorem for the vectors u = (3,0,1,0,4,-1) and v = (-2,5,0,2,-3,-18) **03**

(b) Find the unit vector normal to surface $x^2y+2xz=4$ at the point (2,-2,3). **04**

(c) Verify Green's theorem for $\vec{F}=x^2\hat{i}+xy\hat{j}$ under the square bounded by x=0, x=1, y=0, y=1. **07**

Q.7 (a) Find $curl\vec{F}$ at the point (2, 0, 3), if $\vec{F}=ze^{2xy}\hat{i}+2xy\cos y\hat{j}+(x+2y)\hat{k}$ **03**

(b) Show that the set V=R^3 with the standard vector addition and scalar multiplication defined as c(u_1, u_2, u_3)=(0,0,cu_3) is not vector space. **04**

(c) Use divergence theorem to evaluate $\iint_s (x^3dydz+x^2ydzdx+x^2zdxdz)$ where S is the closed surface consisting of the cylinder $x^2+y^2=a^2$ and the circular discs z=0and z=b. **07**

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