Seat No.: ________ Enrolment No.____________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st/2nd (OLD SYLLABUS) EXAMINATION – SUMMER 2015

Subject code: 110014 **Date: 02/06/2015**

Subject Name: Calculus

Time: 10.30am-01.30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Each question carries equal marks.**

Q.1 **(a)** Do as Directed:

(i) State Reduction formula for $\int_0^{\pi/2} \sin^n x\,dx,\ (n \in N)$. **03**

Hence, evaluate $\int_0^{\pi} x\sin^5 x\,dx.$

(ii) Expand $f(x) = e^x \cos x$ in powers of x up to the terms containing x^4. **04**

(b) Do as Directed:

(i) Determine whether $\int_0^{\pi/2} \sec x\,dx$ converges or diverges. **03**

(ii) Evaluate the limits: **04**

$(i)\ \lim\limits_{x\to 0}\dfrac{x(\cos x - 1)}{\sin x - x};$ $(ii)\ \lim\limits_{x\to 0}\dfrac{2\sqrt{1+x}-2-x}{2\sin^2 x}.$

Q.2 **(a)** Do as Directed:

(i) Find the critical points of $f(x) = 2x^3 - 14x^2 + 22x - 5$. Also, find the points of inflection. **03**

(ii) Using double integrations, find the area of the region common to the parabolas $y^2 = 8x$ and $x^2 = 8y$. **04**

(b) Do as Directed:

(i) Test for convergence or divergence: $\sum_{n=1}^{\infty} \dfrac{n^{10}}{10^n}$. **03**

(ii) Change into polar coordinates and hence evaluate : $\int_0^a \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} y^2\,dy\,dx.$ **04**

Q.3 **(a)** Do as Directed:

(i) Find the local extreme values of the function $f(x,y) = x^3 + y^3 - 3xy$. **03**

(ii) Find the Maclurin's series expansion of $f(x,y) = \sin(2x+3y)$ up to third order derivative terms. **04**

(b) Do as Directed:

(i) State Euler's theorem for function of two variables and apply it to find $x^2 f_{xx} + 2xy\, f_{yx} + y^2 f_{yy}$ for $f(x,y) = \dfrac{x^5 - y^5}{x^2 + 3xy}$. **03**

(ii) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} z\ dz\ dy\ dx.$ **04**

Q.4 **(a)** Do as Directed:

(i) Evaluate $\int_0^{2a} x^{5/2}\sqrt{2ax-x^2}\,dx.$ **03**

(ii) If $x=r\sin\theta\cos\phi;\ y=r\sin\theta\sin\phi;\ z=r\cos\theta$ then find $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$. **04**

(b) Do as Directed:

(i) Find the equations of tangent plane and normal line of the surface $x^2+y^2-z^2=4$ at the point $(1,2,1)$. **03**

(ii) Change into spherical polar coordinates and hence evaluate **04**

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dz\ dy\ dx}{\sqrt{a^2-x^2-y^2-z^2}}\ .$$

Q.5 **(a)** Do as Directed:

(i) Find the area of the loop of the curve $9ay^2=x^2(3a-x)$. **03**

(ii) If $z=f(x,y)$, where $x=s+t;\quad y=s-t$, show that **04**

$$\left(\frac{\partial z}{\partial x}\right)^2-\left(\frac{\partial z}{\partial y}\right)^2=\frac{\partial z}{\partial s}\cdot\frac{\partial z}{\partial t}.$$

(b) Do as Directed:

(i) State Leibniz's rule and hence evaluate $\frac{d}{dx}\left[\int_{x^2}^{\sqrt{x}} \cos(t+1)\ dt\right]$. **03**

(ii) If $z=\sin^{-1}(x-y)$, where $x=3t;\ y=4t^3$, show that $\frac{dz}{dt}=3\left(1-t^2\right)^{-\frac{1}{2}}$. **04**

Q.6 **(a)** Do as Directed:

(i) If $f(x,y)=\frac{x^2y}{x^4+y^2}$, does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? **03**

(ii) Find by double integrations, the volume of the cylinder $x^2+y^2=1$ between the planes $z=0$ and $y+z=2$. **04**

(b) Do as Directed:

(i) Find the asymptotes parallel to the coordinate axes of the curve **03**

$$\left(x^2-4\right)\left(y^2-9\right)=36.$$

(ii) Find the volume by triple integrals of the solid S that is bounded by the plane $2x+3y+4z=12$ and the three coordinate planes. **04**

Q.7 **(a)** Do as Directed:

(i) Discuss the symmetry of the curve $y^2\left(a^2-x^2\right)=x^2\left(a^2+x^2\right)$. **03**

(ii) Change the order of Integrations and hence evaluate: $\int_0^1\int_{2x}^2 e^{y^2}\,dxdy$. **04**

(b) Do as Directed:

(i) Using double integrations, find the area enclosed by the cardioid $r=a(1+\cos\theta)$. **03**

(ii) Find the volume of the solid obtained by rotating the region bounded by $y=x^3$, $y=8$ and $x=0$ about the $y-$axis. **04**

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