Seat No.: _____ Enrolment No._____

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE- SEMESTER 1st / 2nd EXAMINATION (OLD SYLLABUS) – SUMMER - 2017**

**Subject Code: 110014** **Date:01/06/2017**

**Subject Name: Calculus**

**Time:2:30 PM to 05:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)**

1) Find the Taylor's series about $x = 0$ for $f(x) = \frac{1}{1+x^2}$. **03**

2) Find the expansion of $\tan\left(x + \frac{\pi}{4}\right)$ in ascending powers of $x$ up to terms in $x^3$ and find approximately the value of $\tan 43^\circ$. **04**

**(b)**

1) Does the series $\sum_{n=1}^{\infty}\left(1 - \cos\frac{1}{n}\right)$ converges ? Justify. **03**

2) Evaluate: **04**

i. $\lim_{x \to 0} \frac{x^2 - 1 - x}{x^2}$

ii. $\lim_{x \to 0} (\sin x)^{\tan x}$

**Q.2** **(a)**

1) Determine whether the series $\sum_{n=1}^{\infty} \frac{(n+1)!}{n^n}$ converges or diverges. **03**

2) Find the radius and interval of convergence of the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n-1}}{(2n-1)!}$. **04**

**(b)**

1) Determine whether the series $\sum_{n=1}^{\infty} \frac{(n+1)}{2^{n+2}}$ converges or diverges. **03**

2) Discuss the continuity of **04**

$$f(x,y) = \begin{cases} \frac{x^2 - y^2}{x^2 + y^2} & ; (x,y) \neq (0,0) \\ 0 & ; (x,y) = (0,0) \end{cases}$$ at $(0,0)$.

**Q.3** **(a)**

1) Does the improper integral $\int_0^{\infty} e^{-x^2} dx$ converge or diverge ? Justify. **03**

2) Compute the four second order partial derivatives of **04**

$f(x,y) = xy^2 + 3x^2y^3$.

**(b)**

1) Evaluate $\lim_{x \to 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$. **03**

2) Find the maximum and minimum values of $x + y$ on the circle $x^2 + y^2 = 4$. **04**

**Q.4** **(a)**

1) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 14$ at the point $(1,2,3)$. **03**

2) If $u = \sin^{-1}\frac{x^2 + y^2}{x + y}$, show that **04**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \text{suitable terms}$$.

**(b)**

1) Determine the intervals of increasing and decreasing for the function $f(x) = x^3 - 12x - 5$. **03**

2) Evaluate $\iint_D (x + 2y)\,dA$, where $D$ is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$. **04**

**Q.5** **(a)** 1) The region between the curve $y = \sqrt{x}$ ; $0 \le x \le 4$ and the $x$-axis is revolved about the $x$-axis to generate a solid. Find its volume. **03**

2) Find the maximum and minimum values of the function $f(x) = x^2 + y^2 - 3x - 12y + 20$. **04**

**(b)** 1) Evaluate $\int_0^1 \int_0^1 (x+y)\,dy\,dx$ **03**

2) Evaluate the integral $\int_0^1 \int_x^{\sqrt{x}} y\,dy\,dx$ by reversing the order of integration. **04**

**Q.6** **(a)** Trace the curve $ay^2 = 4a^2(2a - x)$; $a > 0$. **07**

**(b)** 1) Evaluate the improper integral $\int_1^\infty \frac{1}{x^2}\,dx$. **03**

2) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} (x^2 + y^2)\,dy\,dx$ by changing into polar coordinates where $a > 0$. **04**

**Q.7** **(a)** Trace the curve $r = a(1 + \cos\theta)$. **07**

**(b)** 1) Evaluate $\int_0^1 \int_0^1 \int_0^1 (x+y+z)\,dz\,dy\,dx$. **03**

2) Expand $e^x \sin y$ in powers of $x$ and $y$ up to second order term. **04**

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