Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st/2nd (NEW SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code:2110014 **Date: 02/06/2015**
Subject Name: Calculus
Time:10.30am-01.00pm **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)**(see below for note on Q.1)**

(a) Attempt the following. **07**

1. Maclaurin series of $\sin x$ is

a) $\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$ b) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ c) $\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$ d) $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

2. The total area enclosed between $y=\sin x$ and $X-$axis in $[0,\pi]$ is

a) 0 b) 4 c) 2 d) 1

3. The value of $\lim_{x\to\infty} x^{1/x}$

a) ∞ b) $-\infty$ c) 1 d) 0

4. How many leaves $r=\sin 5\theta$ has ?

a) 6 b) 2 c) 3 d) 5

5. The sum of the series $\sum_{n=1}^{\infty}\frac{1}{2^n}$ is

a) 0 b) 1 c) -1 d) $1/2$

6. The values of x at which the curves $y=x$ & $y=4x-x^2$ intersects each other are

a) 0 & 4 b) -4 & 4 c) 0 & 3 d) -3 & 3

7. The area enclosed between the curve $r=f(\theta)$ and two rays $\theta=\alpha$ & $\theta=\beta$ is

a) $\int_{\alpha}^{\beta} r^2 d\theta$ b) $\int_{\alpha}^{\beta} r^3 d\theta$ c) $\frac{4\pi}{3}\int_{\alpha}^{\beta} r^3 d\theta$ d) $\frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$

(b) Attempt the following. **07**

1. If $u=\sin^{-1}\frac{x}{y}+\tan^{-1}\frac{y}{x}$ then the value of xu_x+yu_y is

a) u b) $-u$ c) 0 d) 1

2. For an implicit function $f(x,y)=c$ the value of $\frac{dy}{dx}$ is

a) $\frac{f_x}{f_y}$ b) $\frac{f_y}{f_x}$ c) $-\frac{f_x}{f_y}$ d) $-\frac{f_y}{f_x}$

3. For the function $u=\left(x^2+y^2\right)^{1/3}$ the value of $x^2u_{xx}+2xyu_{xy}+y^2u_{yy}$ is

a) $-2u$ b) $-\frac{2u}{3}$ c) $\frac{u}{9}$ d) $-\frac{2u}{9}$

4. If $x=r\cos\theta$ & $y=r\sin\theta$ then $J=\frac{\partial(\mathrm{r},\theta)}{\partial(\mathrm{x},\mathrm{y})}$ is

a) r b) $-r$ c) $\frac{1}{r}$ d) $-\frac{1}{r}$

5. The fundamental period of $\sin 2x$ is

a) π b) 2π c) 4π d) $\frac{\pi}{2}$

6. The focus of parabola $y^2 4ax$ is $(ae,0)$ if

a) $e<1$ b) $e>1$ c) $e=1$ d) $e\neq 1$

7. The function $f(x)=2x^3+3x^2-12x+7$ is decreasing in

a) $[-2,1]$ b) $R-[-2,1]$ c) $[0,2]$ d) $[1,3]$

OR

Q.1 Objective Question (MCQ)(** **14**

1 $\lim_{x\to 2}\frac{x^2-4}{x^2+4}=$

(a) 1 (b) 0 (c) -1/2 (d) None of these

2 $f(x)=\frac{x^2-x}{2x};x\neq 0$

$f(0)=k$

If

and if f is continuous on $x=0$ then $k=$

(a) -1 (b) -1/2 (c) 0 (d) None of these

3 What is the slope of the tangent line to the curve $x+y=xy$ at point $(2,2)$

(a) -1 (b) -2 (c) -3 (d) -4

4 Determine the second derivative of the function $f(x)=x^2\cdot\ln 2x$

(a) $2\ln 2x+3$ (b) $2\ln 2x+\frac{3}{2}$ (c) 0 (d) None of these

5 At a minimum, the second differential function of the form $y=ax^n+bx^{n-1}+.....$ is

(a) Positive (b) Negative (c) Zero (d) Infinite

6 If $y=\frac{3}{x^4}$ then $\int y\,dx$

(a) $\frac{15}{x^5}+c$ (b) $-\frac{1}{x^3}+c$ (c) $\frac{12}{x^3}+c$ (d) None of these

7 Evaluate $y=\int_0^{\frac{\pi}{2}}\sin(4x)\sin(6x)\,dx$

(a) $y=\frac{\pi}{2}$ (b) $y=\frac{\pi}{4}$ (c) $y=\frac{3\pi}{4}$ (d) $y=0$

8 Use matrices to solve the following simultaneous equations

$x+2z=9$

$4x+2y+z=14$

$x+3y+4z=26$

(a) $x=3;y=-1/2;z=3$ (b) $x=-1;y=1;z=6$

(c) $x=1;y=3;z=4$ (d) $x=1;y=4;z=2$

9 Determine the area bounded by

The x axis, the curve $y = \sin 2x$,and the line $x = \frac{\pi}{4}$ and x = π/2

(a) 0.5 (b) 1 (c) 0.25 (d) 1.5

10 Use matrices to solve the following simultaneous equations

$x + 2y = 3$

$2x + 3y = 5$

(a) $x = 2; y = 1$ (b) $x = 1; y = 1$

(c) $x = 2; y = 2$ (d) None of these

11 Determine the area bounded by

The x axis, the curve $y = 2x^2 + x - 6$,and the line $x = 4$ and $x = 6$

(a) 298 (b) 126 (c) $99\frac{1}{3}$ (d) $26\frac{2}{3}$

12 Given the function $f(x) = x^2$ which value of c satisfies the conclusion of mean value theorem on the interval [-4, 5]?

(a) 0 (b) 1 (c) 1/2 (d) None of these

13 Eliminate the parameter in the equations $x = t^2; y = t^4$

(a) $y = x^2 \; for\; x \geq 0$ (b) $y = \sqrt{x} \; for\; x \geq 0$ (c) $y = 2x^2 \; for\; x \geq 0$ (d) None of these

14 At how many places does the curve $x = \cos 3t; y = \sin t$ cross the x-axis?

(a) 2 (b) 1 (c) 3 (d) None of these

Q.2 (a) Show that the $p-series$ $\sum_{n=1}^{\infty} \frac{1}{n^p}$ ($p-$ a real constant) converges if $p > 1$ and diverges if $p \leq 1$. **03**

(b) Define the Geometric series and find the sum of the following series $\sum_{n=1}^{\infty} \frac{3^{n-1} - 1}{6^{n-1}}$. **04**

(c) Attempt the following.

1) Investigate the convergence of the series $\sum_{n=1}^{\infty} \frac{2^n + 5}{3^n}$. **03**

2) Is the series $\sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$ converges or diverges? **04**

Q.3 (a) Define Taylor's series for the function of one variable and using it show that **03**

$$\tan^{-1}(x+h) = \tan^{-1} x + (h \sin\alpha)\frac{\sin\alpha}{1} - (h\sin\alpha)^2 \frac{\sin 2\alpha}{2} + (h\sin\alpha)^3 \frac{\sin 3\alpha}{3} + ----.$$

Where $\alpha = \cot^{-1} x$

(b) Trace the curve $9ay^2 = x(x-3a)^2$. **04**

(c) Attempt the following. **03**

1) Evaluate $\lim_{x \to 0} \left[\frac{a^x + b^x + c^x}{3} \right]^{1/3x}$. **03**

2) Define volume of solid of revolution by washer's method and use it to find the volume of solid generated when the region between the graphs $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about $X-$axis. **04**

Q.4

(a) Define Improper integral of both the kinds. Check the convergence of $\int_0^3 \frac{dx}{\sqrt{9-x^2}}$. **03**

(b) Trace the curve $r^2 = a^2 \cos 2\theta$. **04**

(c) **Attempt the following.**

1) Evaluate $\lim_{x\to 0}\left[\frac{1}{x^2}-\frac{1}{\sin^2 x}\right]$. **03**

2) Define the volume of solid of revolution by disk and use it to find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval $[1,4]$ is revolved about $X-$axis. **04**

Q.5 **(a)** Define Homogenous function of two variables x and y of degree n. Also prove the following Euler's theorem for this homogenous function of degree n. $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=nu$. **03**

(b) If $u=\tan^{-1}\left[\frac{x^3+y^3}{x-y}\right]$ then prove that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\cos 3u\sin u$, **04**

(c) **Attempt the following.**

1) If $x=\rho\sin\phi\cos\theta$, $y=\rho\sin\phi\sin\theta$ and $z=\rho\cos\phi$ then obtain the Jacobean $J=\frac{\partial(\mathrm{x,y,z})}{\partial(\rho,\phi,\theta)}$. **03**

2) In a plane triangle ABC find the extreme values of $\cos A\cos B\cos C$. **04**

Q.6

(a) If $u=\sin^{-1}(x-y)$, $x=3t$, $y=4t^3$ then show that $\frac{du}{dt}=\frac{3}{\sqrt{1-t^2}}$. **03**

(b) If $u=f(r)$ and $r^2=x^2+y^2+z^2$ then show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=f^{''}(r)+\frac{2}{r}f^{'}(r)$, **04**

(c) **Attempt the following.**

1) Find the equations of the tangent plane and normal line to the surface $2xz^2-3xy-4x=7$ at $(1,-1,2)$. **03**

2) Find the minimum value of $x^2+y^2+z^2$, given that $ax+by+cz=p$. **04**

Q.7 **(a)** Evaluate $\iint_R (2x-y^2)\,dA$;over the triangular region R enclosed between the lines $y=-x+1$, $y=x+1$& $y=3$. **03**

(b) Evaluate the integral $\int_0^2\int_{y/2}^1 e^{x^2}\,dxdy$ by changing the order of integration. **04**

(c) **Attempt the following.**

1) Use double integral in polar integral to find the area enclosed by three petaled rose $r=\sin 3\theta$. **03**

2) Use triple integral to find the volume of the solid within the cylinder $x^2+y^2=9$ between the planes $z=1$ and $x+z=1$. **04**

****