

GUJARAT TECHNOLOGICAL UNIVERSITY
BE SEMESTER– 1st / 2nd (OLD SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code: 110008

Date: 02/06/2015

Subject Name: Mathematics-1

Time: 10.30am-01.30pm

Total Marks: 70

Instructions:

1. Attempt any five questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- (a) (i) Verify Cauchy's mean value theorem for $\frac{1}{x}$ and $\frac{1}{x^2} \forall x \in [a, b], a > 0$. **03**
- (ii) Find maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. **04**
- (b) (i) Find the Taylor's series expansion $f(x, y) = x^3 - 2x + 4$ at $a=2$. **03**
- (ii) Evaluate using L'Hospital rule **02**
- $$\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2}$$
- (iii) Evaluate using L'Hospital rule $\lim_{x \rightarrow 1} (1-x) \tan\left(\frac{\pi x}{2}\right)$ **02**
- Q.2**
- (a) (i) Trace the curve $1 \leq r \leq 2, 0 \leq \theta \leq \frac{\pi}{2}$ **04**
- (ii) Discuss convergence of the integral $\int_0^4 \frac{1}{x^{-2}} dx$. **03**
- (b) (i) Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$. **04**
- (ii) State fundamental theorem of integral calculus and hence find $\frac{dy}{dx}$ if **03**
- $$y = \int_1^{x^2} \cos t dt$$
- Q.3**
- (a) (i) Test the convergence of $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$. **04**
- (ii) Does sequence whose n^{th} term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converges? If so find, limit of a_n . **03**
- (b) (i) Show that the series $\frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ is convergent for $p > 0$. **04**
- (ii) Find convergence of sequence $\left\{ \frac{(-1)^n}{2} \right\}$. **03**
- Q.4**
- (a) (i) State and prove Euler's theorem on homogenous functions. **04**

- (ii) If $u = \operatorname{cosec}^{-1} \left(\frac{x+y}{x^2+y^2} \right)$, show that $xu_x + yu_y = \tan u$. 03
- (b) (i) Find the equations of tangent plane and normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point $(2, 1, -3)$. 04
- (ii) The region between the curve $y = \sqrt{x}$, $0 \leq x \leq 4$ and X-axis is revolved about X-axis is generate a solid then find volume. 03
- Q.5** (a) (i) Evaluate $\int_0^1 \int_y^{1+y^2} x^2 y dx dy$ 04
- (ii) If $x = r \cos \theta, y = r \sin \theta, z = z$, calculate $\frac{\partial(x, y, z)}{\partial(r, \theta, z)}$ 03
- (b) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2 + y^2) dx dy$ 04
- (ii) If $u = 2xy, v = x^2 - y^2, x = r \cos \theta, y = r \sin \theta, z = z$, calculate $\frac{\partial(u, v)}{\partial(r, \theta)}$ 03
- Q.6** (a) (i) Find the direction derivative of $f(x, y, z) = 4xz^3 - 3x^2y^2z$ at the point $(2, -1, 2)$. 04
- (ii) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 3)$. 03
- (b) (i) Evaluate using Green's theorem, $\int_c [(x^2 + xy)dx + (x^2 + y^2)dy]$ where c is the surface bounded by $x = \pm 1, y = \pm 1$. 04
- (ii) Prove that vector $\vec{F} = (z + \sin y)i + (x \cos y - z)j + (x - y)k$ is irrotational vector. 03
- Q.7** (a) (i) By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy dy dx$. 04
- (ii) Find $\frac{dy}{dx}$ for $(\cos x)^y = x^{\sin y}$ using partial differentiation. 03
- (b) (i) Does limit of function $\frac{2xy}{x^2 + 3y^2}$ exists at $x=0, y=0$? 04
- (ii) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n \log n}$. 03
