Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER 1st / 2nd EXAMINATION (OLD SYLLABUS) – SUMMER - 2017

Subject Code: 110008 **Date:01/06/2017**

Subject Name: Maths-I

Time: 2:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Attempt the following

(i) If $5x \le f(x) \le 2x^2 + 2, \forall x \in R$ then find $\lim_{x \to 2} f(x)$ **03**

(ii) Find c by the mean value theorem for $f(x) = \log x, x \in [1, e]$ **04**

(b) Attempt the following

(i) Evaluate $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$ **03**

(ii) Expand e^x in powers of $(x - 1)$ by Taylor's series. **04**

Q.2 **(a)** Attempt the following

(i) Trace the curve $y^2(2a - x) = x^3$ **04**

(ii) Check the convergence of $\int_0^1 \frac{dx}{\sqrt{1 - x}}$ **03**

(b) Attempt the following

(i) Check the convergence of $\int_5^\infty \frac{7x + 4}{x^2 + 9} dx$ **04**

(ii) Find the extreme values for $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$ **03**

Q.3 **(a)** Attempt the following

(i) Does the sequence $\left\{ \frac{3}{n + 3} \right\}$ monotone? **04**

(ii) Test the convergence of $\sum_{n=1}^{\infty} \frac{2n + 1}{n^2 + 2n + 1}$ **03**

(b) Attempt the following

(i) Test the convergence of $\sum_{n=1}^{\infty} \frac{3^n n!}{n^n}$ by Ratio Test **04**

(ii) Prove that the series $\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^2 + 1}$ is divergent. **03**

Q.4 **(a)** If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y} \right)$, show that $xu_x + yu_y = \sin 2u$. Also prove that $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 2 \cos 3u \sin u$. **07**

(b) Attempt the following

(i) If $u = f(x - y, y - z, z - x)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **03**

(ii) Find the equations of the tangent plane and normal line to the surface $x^2 + 2y^2 + 3z^2 = 12$ at (1,2,-1). **04**

Q.5 **(a)** Attempt the following

(i) Expand $e^x \log(1+y)$ in powers of *x* and *y*. **04**

(ii) Evaluate $\int_{\pi/2}^{\pi} \int_{1}^{2} x \cos xy\, dy\, dx$ **03**

(b) Find a point on the plane $2x + 3y - z = 5$ which is nearest to the origin. **07**

Q.6 **(a)** Attempt the following

(i) Evaluate $\int_0^{\infty} \int_x^{\infty} e^{-y^2} dy\, dx$ by changing the order of integration. **04**

(ii) Find the volume of the solid generated by revolving the region bounded by $y^2 = x$ and the line *x=1*, about the line *x=1*. **03**

(b) Attempt the following

(i) Evaluate $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z} dz\, dy\, dx$ **04**

(ii) Find the constants *a, b, c* so that $\overline{F} = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k$ is irrotational. **03**

Q.7 **(a)** Attempt the following

(i) Find the area of the loop of the curve $ay^2 = x^2(a - x)$. **04**

(ii) Determine $curl\ \overline{F}$ at the point (2,0,3) given that $\overline{F} = ze^{2xy} i + 2xy \cos yj + (x + 2y)k$. **03**

(b) Attempt the following

(i) Using Green's theorem evaluate the integral $\oint_C [(2x - y^2)dx + (x^2 + y^2)dy]$, where C is the boundary in the *xy*-plane of the area enclosed by the *x* – axis and the semi-circle $x^2 + y^2 + 1$ in the upper half xy-plane. **04**

(ii) Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1,2,-1). **03**

****