Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER– 1st / 2nd (OLD SYLLABUS) EXAMINATION – SUMMER 2015

Subject Code:110009 **Date: 15/06/2015**

Subject Name: Maths-II

Time: 10.30am-01.30pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Solve the system of equations by Gauss Elimination method **04**

$-2x + y - z = 4, \quad x + 2y + 3z = 13, \quad 3x + z = -1$

(2) Use the matrices $A = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 5 \\ 4 & 3 \end{bmatrix}$ to **03**

Verify that $(AB)^{-1} = B^{-1}A^{-1}$

(b) Determine the value of λ so that the system of homogeneous equations **07**

$2x + y + 2z = 0, \quad x + y + 3z = 0, \quad 4x + 3y + \lambda z = 0$ has

(i) Trivial solution (ii) Non- trivial solution.

Q.2 **(a)** (1) Let u = (2,-1, 0, 4) and v = (0,5,-2,1). Evaluate the following terms: **03**

(i) u.v (ii) $||v||$ (iii) d(u,v)

(2) State and prove the Pythagorean theorem in R^n. **04**

(b) (1) Let R^4 be the Euclidean inner product. Find the cosine of the angle between the vectors u = (1,0,1,0) and v = (-3,-3,-3,-3). **02**

(2) Find rank and nullity of the matrix **05**

$$\begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

Q.3 **(a)** (1) Let V be the set of all ordered pairs (x,y) of real numbers over the field R of the real numbers. Check whether V is a vector space over R defined by the operations: $(x_1,y_1) + (x_2,y_2) = (x_1+x_2, y_1+y_2)$ and **04**

$k(x,y) = (k^2x, k^2y)$

(2) Show that s = { (1,0,0),(0,1,0),(0,0,1)} is a basis of R^3. **03**

(b) Define the coordinates of V relative to a basis. Find the coordinates of a polynomial $P = 5 + 11x + 2x^2$ relative to the basis $S=\{1-x, 1+x, 1-x^2\}$ of $p_2(R)$. **07**

Q.4 **(a)** (1) Expand the linearly independent set S= {(1,2,2,1),(1,-1,-1,1)} to be a basis for R^4. **04**

(2) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ then find A^{-4}. **03**

(b) Find the Eigen values and Eigen vectors for the matrix $A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ **07**

Q.5 **(a)** Find matrix P that diagonalizes $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$. Also determine $P^{-1}AP$. **07**

(b) Reduce the quadratic form $3x^2 + 3z^2 + 4xy + 8xz + 8yz$ into canonical form using linear transformation. **07**

Q.6 **(a)** (1) Determine linear transformation $T: R^2 \rightarrow R^3$ such that $T(1,0) = (1,2,3)$ and $T(1, 1) = (0, 1, 0)$. Also find $T(2, 3)$. **04**

(2) Let $T: R^2 \rightarrow R^2$ be defined by $T(x,y) = (2x+3y, 5x+7y)$. Is T one-to-one? If so, find formula for $T^{-1}(x,y)$. **03**

(b) (1) Using induced matrix associated with each transformation determine the new point after applying the transformation to the given point **03**

(i) $x = (1,-2,1)$ reflected about the xy-plane

(ii) $x = (9, 4,-2)$ projected on the x-axis.

(iii) $x = (1,-3)$ rotated 30^0 in the counter clockwise direction.

(2) Let $T : R^2 \rightarrow R^2$ defined by **04**

$$T\left[\begin{pmatrix} x1 \\ x2 \end{pmatrix}\right] = \begin{bmatrix} x1 - 2x2 \\ -x2 \end{bmatrix} \text{ and let } B = \{e_1, e_2\} \text{ and } B' = \left\{\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \end{bmatrix}\right\} \text{ then}$$

using $[T]_{B'} = P^{-1}[T]_B P$, Find $[T]_{B'}$. Where P is transition matrix from B' to B.

Q.7 **(a)** Let R^3 have Euclidean inner product. Transform the basis $S = \{(1, 0, 0), (3, 7, -2), (0, 4, 1)\}$ into an orthonormal basis by using Gram-Schmidt process. **07**

(b) (1) Find the least squares solution of the linear system AX = b given by $x_1 + x_2 = 7$, $-x_1 + x2 = 0$, $-x_1 + 2x_2 = -7$. Find the orthogonal projection of b on the column space of A. **05**

(2) Show that matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is an orthogonal matrix. **02**

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