Seat No.: ________ Enrolment No.____________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE SEMESTER– 1st/2nd (NEW SYLLABUS) EXAMINATION – SUMMER 2015**

**Subject Code: 2110015** **Date:15/06/2015**

**Subject Name: Vector Calculus and Linear Algebra**

**Time:10.30am-01.00pm** **Total Marks: 70**

**Instructions:**

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)** Answer the following MCQ **07**

**1.** The angle between $\bar{u} = (-1,1,2,-2)$ and $\bar{v} = (2,-1,-1,3)$ is_____

(a) 3 rad (c) 2.68 rad
(b) 3.68 rad (d) 2 rad

**2.** Which of the following matrix is orthogonal?

(a) $\begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}$ (c) $\begin{bmatrix} 1/9 & \sqrt{8}/9 \\ \sqrt{5}/9 & 2/9 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

**3.** If a square matrix $A$ is involutory then $A^2 =$ ________

(a) $A$ (c) $A^T$
(b) $I$ (d) $A^{-1}$

**4.** A homogeneous system of equations have at least _____solutions

(a) 1 (c) 3
(b) 2 (d) 4

**5.** The rank of $\begin{bmatrix} 1 & 3 & 6 \\ 2 & 1 & 6 \end{bmatrix}$ is

(a) 1 (c) 3
(b) 2 (d) No rank

**6.** If 3 is the eigen value of A then the eigen value of $A + 3I$ is

(a) 9 (c) 0
(b) 6 (d) 27

**7.** Which of the following is a subspace of $R^2$ under standard operations

(a) $R^3$ (c) $M_{22}$
(b) $P_2$ (d) $R$

**(b)** Attempt the following MCQ **07**

**1.** If R is a vector space then which of the following is a trivial subspace of R?

(a) $\{\bar{0}\}$ (c) $\{\bar{0},\bar{1}\}$
(b) R (d) $\{\bar{1}\}$

**2.** The set $S = \{1, x, x^2\}$ spans which of the following?

(a) $R^2$ (c) $P_2$
(b) $M_{22}$ (d) $R$

**3.** The dimension of the solution space of $x - 3y = 0$ is___________

(a) 1 (c) 3
(b) 2 (d) 4

**4.** The mapping $T : R^3 \to R^3$ defined by $T(v_1, v_2, v_3) = (v_1, v_2, 0)$ is called as

(a) Reflection (c) Rotation
(b) Magnification (d) Projection

5. If $\langle \bar{u},\bar{v}\rangle = 9u_1v_1+4u_2v_2$ is the inner product on $R^2$ then it is generated by

(a) $\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}$ (c) $\begin{bmatrix}0 & 2\\ 3 & 0\end{bmatrix}$

(b) $\begin{bmatrix}3 & 0\\ 0 & 2\end{bmatrix}$ (d) $\begin{bmatrix}0 & 3\\ 2 & 0\end{bmatrix}$

6. The divergence of $\bar{F} = xyzi + 3x^2yj + (xz^2 - y^2z)k$ at (2,-1,1) is

(a) $yz + 3x + 2xz$ (c) $yz + 3x^2 + (2xz - y^2)$

(b) $yz + xy$ (d) $xy - yz$

7. If $\bar{F}$ is conservative field then $curl\bar{F} =$ __________

(a) $i$ (c) $k$

(b) $j$ (d) $\bar{0}$

**Q.2** **(a)** Is $A = \begin{bmatrix}2 & 2 & 1\\ -2 & 1 & 2\\ 1 & -2 & 2\end{bmatrix}$ orthogonal? If not, can it be converted into an orthogonal matrix? **03**

**(b)** Solve the following system: $x+y+z=3, x+2y-z=4, x+3y+2z=4.$ **04**

**(c)(i)** Find the rank of the matrix $A = \begin{bmatrix}1 & 2 & 4 & 0\\ -3 & 1 & 5 & 2\\ -2 & 3 & 9 & 2\end{bmatrix}$ **03**

**(ii)** Find the inverse of $A = \begin{bmatrix}3 & -3 & 4\\ 2 & -3 & 4\\ 0 & -1 & 1\end{bmatrix}$ using row operations. **04**

**Q.3** **(a)** Does $W = \{(x,y,z)/x^2+y^2+z^2=1\}$ a subspace of $R^3$ with the standard operations? **03**

**(b)** Find the projection of $\bar{u} = (1,-2,3)$ along $\bar{v} = (2,5,4)$ in $R^3$. **04**

**(c)** Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix}3 & 1 & 4\\ 0 & 2 & 6\\ 0 & 0 & 5\end{bmatrix}$ **07**

**Q.4** **(a)** Find the least square solution for the system $4x_1-3x_2=12, 2x_1+5x_2=32, 3x_1+x_2=21.$ **03**

**(b)** Determine the dimension and basis for the solution space of the system $x_1+2x_2+x_3+3x_4=0, 2x_1+5x_2+2x_3+x_4=0, x_1+3x_2+x_3-x_4=0.$ **04**

**(c)** Check whether $V = R^2$ is a vector space with respect to the operations $(u_1,u_2)+(v_1,v_2) = (u_1+v_1-2, u_2+v_2-3)$ and $\alpha(u_1,u_2) = (\alpha u_1+2\alpha-2, \alpha u_2-3\alpha+3), \alpha \in R$. **07**

**Q.5** **(a)** Consider the inner product space $P_2$. Let $\overline{p_1} = a_2x^2+a_1x+a_0$ and $\overline{p_2} = b_2x^2+b_1x+b_0$ are in $P_2$, where $\langle \overline{p_1}, \overline{p_2}\rangle = a_2b_2+a_1b_1+a_0b_0$. Find the angle between $2x^2-3$ and $3x+5$. **03**

**(b)** Find the rank and nullity of the matrix $A=\begin{bmatrix}2 & 0 & -1\\ 4 & 0 & -2\\ 0 & 0 & 0\end{bmatrix}$ and verify the dimension theorem. **04**

**(c)** Let $T:R^2 \to R^3$ be the linear transformation defined by $T(x_1,x_2)=(x_2,-5x_1+13x_2,-7x_1+16x_2)$. Find the matrix for the transformation T with respect to the bases $B=\{(3,1),(5,2)\}$ for $R^2$ and $B'=\{(1,0,-1),(-1,2,2),(0,1,2)\}$ for $R^3$. **07**

**Q.6** **(a)** Find the arc length of the portion of the circular helix $\bar{r}(t)=\cos ti+\sin tj+tk$ from $t=0$ to $t=\pi$. **03**

**(b)** A vector field is given by $\overline{F}=(x^2+xy^2)i+(y^2+x^2y)j$. Show that $\overline{F}$ is irrotational and find its scalar potential. **04**

**(c)(i)** Let $R^3$ have the Euclidean inner product. Transform the basis {(1,1,1),(1,-2,1),(1,2,3)} into an orthogonal basis using Gram-Schmidt process. **05**

**(ii)** Express the following quadratic form in matrix notation: **02**

$2x^2+5y^2-6z^2-2xy-yz+8zx.$

**Q.7** **(a)** If $\phi = xyz-2y^2z+x^2z^2$, find $div(grad\phi)$ at the point (2,4,1). **03**

**(b)** Use Green's theorem to evaluate $\oint_C [x^2ydx+y^3dy]$, where C is the closed path formed by $y=x$ and $y=x^3$ from (0,0) to (1,1). **04**

**(c)** Verify Stoke's theorem for $\overline{F}=(y-z+2)i+(yz+4)j-xzk$ over the surface of the cube *x=0, y=0, z=0, x=2, y=2, z=2* above the *xy* - plane.(that is open at bottom) **07**

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