

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-III • EXAMINATION – SUMMER • 2014

Subject Code: 130002

Date: 02-06-2014

Subject Name: Advanced Engineering Mathematics

Time: 02.30 pm - 05.30 pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Solve $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ **03**
- (b) $[(x+1)e^x - e^y]dx - xe^y dy = 0, y(1) = 0.$ **04**
- (c) Determine the series solution for the differential equation $y'' + y = 0$ about $x_0 = 0.$ **07**
- Q.2** (a) (i) Solve $(D^2 + 5D + 6)y = e^x.$ **03**
- (ii) Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by method of variation of parameter. **04**
- (b) Solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ by the method of separation of variable. **07**
- OR**
- (b) Solve in series the equation $\frac{d^2y}{dx^2} + xy = 0.$ **07**
- Q.3** (a) Find the Fourier series for $f(x) = e^{-x}, 0 < x < 2\pi.$ **07**
- (b) Find the Fourier series expansion for $f(x)$, if $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ **07**
- Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}.$
- OR**
- Q.3** (a) Find the Fourier series expansions of **07**
- (i) $f(x) = x, -\pi < x < \pi, f(x+2\pi) = f(x)$
- (ii) $f(x) = x^2, -l < x < l.$
- (b) Express $f(x) = x$ as a **07**
- (i) half range sine series in $0 < x < 2$
- (ii) half range cosine series in $0 < x < 2.$
- Q.4** (a) (1) Prove that $L(\sinh at) = \frac{a}{s^2 - a^2}$ for $s > |a|.$ **03**
- (2) Find the Laplace transforms of (i) $\sin 2t \sin 3t$ (ii) $e^{-3t}(2 \cos 5t - 3 \sin 5t)$ **04**
- (b) Evaluate : (i) $L^{-1}\{\ln(1 + \frac{w^2}{s^2})\}$ (ii) $L^{-1}\{\frac{5s+3}{(s-1)(s^2+2s+5)}\}$ **07**
- OR**
- Q.4** (a) Apply convolution theorem to evaluate $L^{-1}\left(\frac{s}{(s^2 + a^2)^2}\right)$ **03**
- (b) Use Laplace transform method to solve $y'' + a^2 y = k \sin at$ **04**

- (c) If $L(f(t)) = \overline{f(s)}$ and if $L\left\{\frac{f(t)}{t}\right\}$ exists then prove that **07**
- $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \overline{f(s)} ds$. Also find $L\left\{\frac{\sin 2t}{t}\right\}$
- Q.5 (a)** (1) Form partial differential equation of $f(x+y+z, x^2+y^2+z^2)=0$, where **03**
f is an arbitrary function.
- (2) Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$ **04**
- (b)** (1) Solve $p(1+q) = qz$ **07**
(2) Solve $p^2 + q^2 = x + y$
- OR**
- Q.5 (a)** Define following terms (i) Beta function (ii) Sinusoidal function **04**
- (b)** Form partial differential equation $z = (x-2)^2 + (y-3)^2$ **03**
- (c)** Find the Fourier integral representation of the function $f(x) = \begin{cases} 2, & |x| < 2 \\ 0, & |x| > 2 \end{cases}$ **07**
