

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-III (OLD) - EXAMINATION – SUMMER 2018

Subject Code: 130002

Date: 18/05/2018

Subject Name: Advanced Engineering Mathematics (New)

Time: 10:30 AM to 01:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) (i) Find the General Solution of the differential equation $(x^2 - y^2)dy = 2xy dx$. **03**

(ii) Find the General Solution of the differential equation **04**

$$(1 + y^2)dx = (e^{-\tan^{-1}y} - x)dy.$$

(b) (i) Define Ordinary Point of the differential equation $y'' + P(x)y' + Q(x)y = 0$. **03**
Give an example of it.

(ii) Solve the differential equation $y'' + y = 0$ by the Power Series Method. **04**

Q.2 (a) Solve the differential equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0, u(x, y) = 4e^{-x}$ by the **07**

Method of Separation of Variables

(b) Find the Series Solution of the Differential Equation **07**
 $2x^2 y'' + (2x^2 - x)y' + y = 0.$

OR

(b) (i) Find the Particular Solution of the Differential Equation $y'' + 25y = \cos 7x$. **03**
by using the Method of Undetermined Co-efficients.

(ii) Using Method of Variation of Parameters solve the differential equation **04**
 $y'' + y = \cot x.$

Q.3 (a) Express the function $f(x) = x - x^2$ as a Fourier Series in the interval $[-\pi, \pi]$. **07**

(b) Find the Fourier Series to represent the function defined as follows **07**

$$f(x) = \begin{cases} \pi x & ; 0 \leq x \leq 1 \\ \pi(2 - x) & ; 1 \leq x \leq 2 \end{cases}$$

OR

Q.3 (a) Express the function $f(x) = \frac{(\pi - x)^2}{4}$ as a Fourier Series in the interval **07**
 $[0, 2\pi]$.

(b) Find Half Range Cosine Series for the function $f(x) = e^x$ in interval $[0, 1]$. **07**

Q.4 (a) (i) Prove that $L\{\cosh at\} = \frac{s}{s^2 - a^2}$. **03**

(ii) State First Shifting Theorem and using it compute $L\{e^{3t}(2\sin 4t - 3\cos 4t)\}$. **04**

(b) (i) Find the Inverse Laplace Transform of the function: **03**

$$F(s) = \frac{1}{(s + \sqrt{2})(s - \sqrt{3})}.$$

(ii) Find the Inverse Laplace Transform of the function: **04**

$$F(s) = \frac{s^2}{(s^2 + 25)(s^2 + 49)}.$$

OR

Q.4 (a) Using Laplace Transform solve the differential equation: **07**

$$y'' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1.$$

(b) (i) Find the Laplace Transform of the function $f(t) = t \sin t$. **03**

(ii) Find the Laplace Transform of the function $f(t) = \frac{\sin t}{t}$. **04**

Q.5 (a) Define following functions: **07**

- (i) Beta Function;
- (ii) Gamma Function;
- (iii) Error Function & Complementary Error Function.

(b) Find the Fourier Transform of the function f defined as follows: **07**

$$f(x) = \begin{cases} xe^{-x} & x > 0; \\ 0 & x < 0. \end{cases}$$

OR

Q.5 (a) (i) Form the Partial Differential Equation by eliminating arbitrary constants **03**

$$\text{from } z = (x^2 + a)(y^2 + b).$$

(ii) Solve: $\frac{\partial^2 z}{\partial x \partial y} = x^2 + y^2$. **04**

(b) (i) Solve: $p^2 + q^2 = 1$. **07**

(ii) Solve: $(y - z)p + (z - x)q = x - y$.
