

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER- III(OLD) EXAMINATION – SUMMER 2019****Subject Code: 130002****Date: 30/05/2019****Subject Name: Advanced Engineering Mathematics****Time: 02:30 PM TO 05:30 PM****Total Marks: 70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** (i) Solve $3e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$ **03**
 (ii) Solve $y' - (1 + 3x^{-1})y = x + 2$; $y(1) = e - 1$ **04**

- (b) Find the Power series solution of the differential equation $y'' = y'$. **07**

- Q.2 (a)** Using the method of separation of variables solve $u_{xx} = 16 u_y$. **07**
 (b) Find the series solution of the differential equation by Frobenius method **07**

$$x \frac{d^2 y}{dx^2} + \frac{dy}{dx} - y = 0$$

OR

- (b) (i) Solve $y'' + 4y = 8 \cos 2x$, $y(0) = 0$, $y'(0) = 2$ **03**
 (ii) Solve $y'' - 4y' - 12y = 7e^{-7x}$ by method of undetermined coefficients. **04**

- Q.3 (a)** Find the Fourier series for the function $f(x) = x^2 + x$, $-\pi \leq x \leq \pi$. **07**

- (b) Find the Fourier series of the function **07**

$$f(x) = \begin{cases} -\pi, & 0 < x < \pi \\ x - \pi, & \pi < x < 2\pi \end{cases}$$

OR

- Q.3 (a)** Find the Fourier series with period 3 to represent $f(x) = 2x - x^2$ in the range $(0, 3)$. **07**

- (b) Find the half range Fourier cosine series of the function $f(x) = c - x$ in interval $(0, c)$ with period $2c$. **07**

- Q.4 (a)** (i) Find the Laplace transform of $e^{-t}(4t^3 + 3\cos 2t + 2e^{-2t})$ **03**
 (ii) Prove that **04**

$$L(\sin at) = \frac{a}{s^2 + a^2} \text{ and } L(\cos at) = \frac{s}{s^2 + a^2}$$

$s > 0$, where a is a constant.

- (b) Find the Inverse Laplace transform of **07**

$$(1) \frac{s+3}{(s^2+1)(s^2+9)} \quad (2) \frac{2s+3}{s^2-2s+5}$$

OR

- Q.4 (a)** (i) Find the Laplace transform of **03**

$$e^{-2t} \int_0^t t \cos t \, dt$$

- (ii) Find the Inverse Laplace transform of **04**

$$\frac{1 + e^{-\frac{\pi}{2}s}}{s^2 + 4}$$

- (b) Using Laplace transform solve the differential equation $y'' + 6y = 1$, $y(0) = 2$, $y'(0) = 0$ **07**
- Q.5 (a)** (i) Form Partial differential equation by eliminating the arbitrary function from the equation **03**
- $$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$
- (ii) Define the following: (1) Beta function (2) Dirac's Delta Function **04**
- (b) Express the function as a Fourier Integral **07**
- $$f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$
- OR**
- Q.5 (a)** (i) Solve : $p + q = pq$ **03**
- (ii) Solve: $x(y^2 - z^2)p + y(z^2 - x^2)q = z(x^2 - y^2)$. **04**
- (b) Solve the following: **07**
- (i) $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$
- (ii) $(D - D' - 1)(D - D' - 2)z = e^{2x - y}$
