Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III • EXAMINATION – WINTER • 2014

Subject Code: 130002 **Date: 06-01-2015**

Subject Name: Advanced Engineering Mathematics

Time: 02.30 pm - 05.30 pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Find the differential equation of the family of circles of radius r whose centre lies on the x-axis. **04**

(2) Solve $\frac{dy}{dx} + 2y\,tanx = sinx$ **03**

(b) Find the series solution of $y' - 2xy = 0$ **07**

Q.2 **(a)** Solve by method of separation of variables $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 2(x+y)u$ **07**

(b) Solve in series the differential equation $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ **07**

OR

(b) (1) Solve the Initial Value Problem $y'' - 9y = 0$; $y(0) = 2$, $y'(0) = -1$ **03**

(2) Solve $(D^2 + 16)y = x^4 + e^{3x} + \cos 3x$ **04**

Q.3 **(a)** (1) Find the Laplace transform of $f(t) = 0$; $0 \le t \le 3$

$= 4$; $t \ge 3$ **03**

(2) Prove that $L(\sin h\, kt) = \frac{k}{s^2 - k^2}$ **04**

(b) (1) Find $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$ **03**

(2) Find $L^{-1}\left\{\frac{3s^2+2}{(s+1)(s+2)(S+3)}\right\}$ **04**

OR

Q.3 **(a)** If L{f(t)}= F(s) then show that $L\{t^n f(t)\} = (-1)^n \frac{d^n}{ds^n}\{F(s)\}$; n =1,2,3…. and use this result find $L(t^2 \sin wt)$ **07**

(b) Solve the differential equation by Laplace Transform **07**

$y'' + y = \sin 2t$; $y(0) = 2$, $y'(0) = 1$

Q.4 **(a)** Find the Fourier series of $f(x) = x + |x|$; $-\pi < x < \pi$ **07**

(b) Find the Fourier expansion $f(x) = x^2 - 2$; $-2 \le x \le 2$ **07**

OR

Q.4 **(a)** Obtain the cosine series for the function $f(x) = e^x$ in the range $(0,l)$ **07**

(b) Find the Fourier series for the periodic function f(x) **07**

$f(x) = -k$; if $-\pi < x < 0$ $f(x + 2\pi) = f(x)$

$= k$; if $0 < x < \pi$

Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots. = \frac{\pi}{4}$

Q.5 **(a)** (1) Define the following terms: **04**

(i) Beta function

(ii) Heaviside’s function

(2) Eliminate the arbitrary function from the equation **03**

$z = xy + f(x^2 + y^2)$

(b) Using Fourier integral representation, show that **07**

$$\int_0^\infty \frac{\cos x\lambda + \lambda \sin x\lambda}{1+\lambda^2}\, d\lambda = 0 \quad ; \quad x < 0$$
$$= \pi/2 \quad ; \quad x = 0$$
$$= \pi e^{-x} \quad ; \quad x > 0$$

OR

Q.5 **(a)** (1) Solve $(y + z)\, p + (z + x)\, q = x + y$ **04**

(2) Solve $p^2 + q^2 = npq$ **03**

(b) (1) Solve $\frac{\partial^3 z}{\partial x^2 \partial y} = \cos(2x+3y)$ **04**

(2) Solve $(D^2 + 10DD' + 25D'^2)\, z = e^{3x+2y}$. **03**

****