

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER-III EXAMINATION – WINTER 2015

Subject Code:130002

Date:31/12/2015

Subject Name: Advanced Engineering Mathematics

Time: 2:30pm to 5:30pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1** (a) Attempt the following
- (i) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$. 4
- (ii) Solve $(e^y + 1) \cos x dx + e^y \sin x dy = 0$. 3
- (b) Attempt the following
- (i) Solve $(D^2 - 5D + 6)y = x + e^{4x}$. 4
- (ii) Define triangular wave function and draw its graph. 3
- Q.2** (a) Attempt the following
- (i) Solve $(D^2 - 8D + 9)y = 40 \sin 5x$. 4
- (ii) Find ordinary and singular points for $2x(x-2)^2 y'' + 3xy' + (x-2)y = 0$. 3
- (b) Attempt the following
- (i) Solve $(1+x)^2 y'' + (1+x)y' + y = 4 \cos \log(1+x)$. 5
- (ii) Define beta function. Find $B(5,4)$. 2
- OR**
- (b) Find the power series solution about $x=0$ of $y'' + xy' + x^2 y = 0$. 07
- Q.3** (a) Express $f(x) = x \cos x$ as a Fourier series in $(-\pi, \pi)$. 07
- (b) Find Fourier series expansion of the function given by 07
- $$f(x) = \begin{cases} 0, & -2 < x < 0 \\ 1, & 0 < x < 2 \end{cases}$$
- OR**
- Q.3** (a) If $f(x) = \begin{cases} x, & 0 \leq x \leq \pi \\ 2\pi - x, & \pi \leq x \leq 2\pi \end{cases}$, find the Fourier series. 07
- (b) Prove that $\frac{l}{2} - x = \frac{l}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2n\pi x}{l}, 0 < x < l$. 07
- Q.4** (a) Attempt the following
- (i) Evaluate $L^{-1} \left\{ \log \left(\frac{s+4}{s+3} \right) \right\}$. 4
- (ii) Find $L \{ t^2 \sin 4t \}$. 3
- (b) Attempt the following
- (i) Find the Laplace transform of the periodic function defined by 4
- $$f(t) = \frac{t}{2}, 0 < t < 3, \quad f(t+3) = f(t).$$

(ii) Find Laplace inverse of $\frac{1}{s(s+a)^3}$. 3

OR

Q.4 (a) Attempt the following
(i) Use convolution theorem to find $L^{-1}\left\{\frac{1}{s(s^2+a^2)}\right\}$. 4

(ii) Find $L\left\{\frac{1-\cos 2t}{t}\right\}$. 3

(b) Attempt the following
(i) Solve by Laplace transform: $\frac{dy}{dt} - 2y = 4$, given that $t = 0, y = 1$. 5

(ii) Find $L(t^2 * \cos t)$. 2

Q.5 (a) Attempt the following
(i) Derive partial differential equation by eliminating a and b from $z = ax + by + ab$. 3
(ii) Find the complete integral of $q = pq + p^2$. 4
(b) Solve the p.d.e. $u_{xx} = 16u_y$ by using separation of variables method. 07

OR

Q.5 (a) Attempt the following
(i) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$. 3

(ii) Solve $2r + 5s + 2t = 0$. 4

(b) Using Fourier integral show that 07

$$\int_0^\infty \frac{1 - \cos \pi \lambda}{\lambda} \sin x \lambda d\lambda = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$
