Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (OLD) EXAMINATION – WINTER 2017

Subject Code:130002 Date:09/11/2017

Subject Name: Advanced Engineering Mathematics

Time: 10:30 AM to 01:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)(1)**	Obtain the differential equations of all parabolas whose axes are parallel to the axis of y.	**03**
	(2)	Prove that $\int_0^2 x^4\left(8-x^3\right)^{\frac{1}{3}}dx = \frac{16}{3}\beta\left(\frac{5}{3},\frac{2}{3}\right)$	**04**
	(b)	Obtain the solution of $(x^2y-2xy^2)dx-(x^3-3x^2y)dy=0$	**07**
Q.2	**(a)(1)**	Find the general solution of $\frac{d^4y}{dx^4}-18\frac{d^2y}{dx^2}+81y=0$	**03**
	(2)	Find the particular solution of $\left\{(x+1)e^x-e^y\right\}dx-xe^y dy=0, y(1)=0$	**04**
	(b)	Obtain the general solution of $x^2y''-4xy'+6y=21x^{-4}$	**07**
		OR	
	(b)	Find the general solution of $\left(D^2-4D+3\right)y=\sin 3x\cos 2x$	**07**
Q.3	**(a)(1)**	Find the Laplace transform of $f(t)=e^t$.	**03**
	(2)	Find the Inverse Laplace transform of $\frac{1}{s^3\left(s^2+a^2\right)}$	**04**
	(b)	Solve the differential equation $\frac{d^2y}{dt^2}+4y=f(t),\ y(0)=0,\ y'(0)=1$, using Laplace transform where $f(t)=H(t-2)$	**07**
		OR	
Q.3	**(a)**	Obtain the solution of the differential equation $\frac{d^2y}{dx^2}+y=0$, using the power series method.	**07**
	(b)	Obtain the general solution of $(D-2)^3y=17e^{2x}$, using the method of undetermined coefficients.	**07**
Q.4	**(a)(1)**	Discuss the nature of points -1 and 0 for $xy''+(\sin x)y=0$	**03**
	(2)	Find the Fourier series representation of the given periodic function $f(t)=\begin{cases}-1 \ if\ -\pi<t<0 \\ 1 \ if\ 0<t<\pi\end{cases}\quad,\quad f(t)=f(t+2\pi)$	**04**

(b) Obtain the Fourier series representation of the periodic function $f(x)$ of period $p = 2\pi$ and also sketch $f(x)$, where $f(x) = \begin{cases} x & if\ \frac{-\pi}{2} < x < \frac{\pi}{2} \\ 0 & if\ \frac{\pi}{2} < x < \frac{3\pi}{2} \end{cases}$ **07**

OR

Q.4 (a)(1) Find the convolution of the functions t and e^t. **03**

(2) Find the indicial roots of the given differential equation $4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$ **04**

(b) Using the method of Frobenius, obtain the two linearly independent solutions about $x=0$ for the equation $x^2y'' + x(x-1)y' + (1-x)y = 0$ **07**

Q.5 (a)(1) Find the Fourier cosine integral of $f(x) = e^{-kx}$ $(x > o\ ,\ k > 0)$. **03**

(2) Find the solution of $y^2p - xyq = x(z - 2y)$. **04**

(b) Find the solution of $2\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = \sin(2x + y)$. **07**

OR

Q.5 (a)(1) Obtain the partial differential equation by eliminating the arbitrary function from the equation $z = xy + f\left(x^2 + y^2\right)$. **03**

(2) Find general solution of $y''' - 3y'' + 3y' - y = 4e^t$ **04**

(b) Using the method of separation of variables, find the solution of the partial differential equation $u_{xx} = 16u_y$ **07**

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