Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (NEW)EXAMINATION – SUMMER 2015

Subject Code:2130002 **Date: 06/06/2015**

Subject Name:Advanced Engineering Mathematics

Time:02.30pm-05.30pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Solve the differential equation $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$. **04**

(2) Solve the differential equation $ye^x dx + (2y + e^x)dy = 0$. **03**

(b) Find the series solution of $(1 + x^2)y'' + xy' - 9y = 0$. **07**

Q.2 **(a)** (1)Solve the differential equation using the method of variation of parameter $y'' + 9y = \sec 3x$. **04**

(2) Solve the differential equation $(D^2 - 2D + 1)y = 10e^x$. **03**

(b) Using the method of separation of variables, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$; $u(x,0) = 6e^{-3x}$. **07**

OR

(b) Find the series solution of $2x(x-1)y'' - (x+1)y' + y = 0;\quad x_0 = 0$ **07**

Q.3 **(a)** Find the Fourier Series for $f(x) = \begin{cases} \pi + x; & -\pi < x < 0 \\ \pi - x; & 0 < x < \pi \end{cases}$ **07**

(b) (1) Find the Half range Cosine Series for $f(x) = (x-1)^2; 0 < x < 1$. **04**

(2) Find the Fourier sine series for $f(x) = e^x; 0 < x < \pi$. **03**

OR

Q.3 **(a)** Find the Fourier Series for $f(x) = \begin{cases} -\pi; & -\pi < x < 0 \\ x - \pi; & 0 < x < \pi \end{cases}$. **07**

(b) (1) Find the Fourier cosine series for $f(x) = x^2; 0 < x < \pi$. **04**

(2) Find the Fourier sine series for $f(x) = 2x; 0 < x < 1$. **03**

Q.4 **(a)** (1) Prove that (i) $L(e^{at}) = \frac{1}{s-a}; s > a$ (ii) $L(\sinh at) = \frac{a}{s^2 - a^2}$. **04**

(2) Find the Laplace transform of $t \sin 2t$. **03**

(b) (1) Using convolution theorem, obtain the value of $L^{-1}\left\{\frac{1}{s(s^2+4)}\right\}$. **04**

(2) Find the inverse Laplace transform of $\frac{1}{(s-2)(s+3)}$. **03**

OR

Q.4 **(a)** Solve the initial value problem using Laplace transform: $y'' + 3y' + 2y = e^t$, $y(0) = 1$, $y'(0) = 0$. **07**

(b) (1) Find the Laplace transform of $f(t) = \begin{cases} 0\ ; & 0 < t < \pi \\ \sin t; & t \ge \pi \end{cases}$. **04**

(2) Evaluate $t * e^t$. **03**

Q.5 **(a)** Using Fourier integral representation prove that

$$\int_0^\infty \frac{\cos\lambda x + \lambda \sin\lambda x}{1+\lambda^2}\, d\lambda = \begin{cases} 0 & if \quad x < 0 \\ \frac{\pi}{2} & if \quad x = 0 \\ \pi e^{-x} & if \quad x > 0 \end{cases}.$$ **07**

(b) (1) Form the partial differential equation by eliminating the arbitrary functions from $f\left(x + y + z, x^2 + y^2 + z^2\right) = 0$. **04**

(2) Solve the following partial differential equation $(z - y)p + (x - z)q = y - x$. **03**

OR

Q.5 **(a)** A homogeneous rod of conducting material of length 100 cm has its ends kept at zero temperature and the temperature initially is

$$u(x,0) = \begin{cases} x & ; \quad 0 \le x \le 50 \\ 100 - x; & 50 \le x \le 100 \end{cases}$$ **07**

Find the temperature $u(x,t)$ at any time.

(b) (1) Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$. **04**

(2) Solve $p - x^2 = q + y^2$. **03**

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