

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-III (NEW) EXAMINATION – SUMMER 2019

Subject Code: 2130002

Date: 30/05/2019

Subject Name: Advanced Engineering Mathematics

Time: 02:30 PM TO 05:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

MARKS

Q.1 (a) Solve $(x + y - 2)dx + (x - y + 4)dy = 0$ 03

(b) Solve $(1 + y^2) + (x - e^{-\tan^{-1}y})\frac{dx}{dy} = 0$ 04

(c) Expand $f(x) = |\cos x|$ as a Fourier series in the interval $-\pi < x < \pi$ 07

Q.2 (a) Define unit step function and unit impulse function. Also sketch the graphs. 03

(b) Solve $\left(\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y\right) = 4\sin 2x$ 04

(c) Find the series solution of $y'' + xy' + y = 0$ about the ordinary point $x = 0$. 07

OR

(c) Find the Fourier series expansion for $f(x)$, if 07

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}, \quad \text{Also deduce that}$$

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

Q.3 (a) Using Fourier integral representation, show that 03

$$\int_0^\infty \frac{1 - \cos \pi \omega}{\omega} \sin \omega x d\omega = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

(b) Solve $\left(\frac{d^2y}{dx^2} + y\right) = x^2 \sin 2x$ 04

(c) Solve by method of variation of parameters $\left(\frac{d^2y}{dx^2} + 9y\right) = \frac{1}{1 + \sin 3x}$ 07

OR

Q.3 (a) Find Laplace transform of $te^{at} \sin at$ 03

(b) Solve $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 5e^x - \sin 2x$ 04

(c) Solve

$$x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + 4y = \cos(\log x) + x \sin(\log x) \quad 07$$

Q.4 (a) Find the orthogonal trajectories of the curve $y = x^2 + c$ **03**

(b) Find the Laplace transform of (i) $\cos(at + b)$ **04**

(ii) $\sin^2 3t$

(c) State convolution theorem and apply it to evaluate **07**

$$L^{-1} \left(\frac{s^2}{(s^2 + 4)^2} \right)$$

OR

Q.4 (a) Solve $\frac{d^3 y}{dx^3} - 3 \frac{d^2 y}{dx^2} + 4y = 0$ **03**

(b) Find Half range cosine series for $f(x) = (x-1)^2$ in the interval $0 < x < 1$ **04**

(c) Solve $y'' + 4y' + 3y = e^{-t}$, $y(0) = y'(0) = 1$ using Laplace transform. **07**

Q.5 (a) Form the partial differential equation by eliminating the arbitrary constants from $z = ax + by + a^2 + b^2$ **03**

(b) Solve $(y - z)p + (x - y)q = z - x$ **04**

(c) Solve $3 \frac{\partial u}{\partial x} + 2 \frac{\partial u}{\partial y} = 0$, where $u(x, 0) = 4e^{-x}$ using the method of separation of variables. **07**

OR

Q.5 (a) Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$ **03**

(b) Solve $\log \left(\frac{\partial^2 z}{\partial x \partial y} \right) = x + y$. **04**

(c) A bar with insulated sides is initially at temperature $0^\circ C$, throughout. The end $x = 0$ is kept at $0^\circ C$ and heat is suddenly applied at the end $x = l$ so that $\frac{\partial u}{\partial x} = A$ for $x = l$, where A is a constant. Find the temperature function. **07**
