

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER-III (New) EXAMINATION – WINTER 2015

Subject Code: 2130002

Date: 31/12/2015

Subject Name: Advanced Engineering Mathematics

Time: 2:30pm to 5:30pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 Answer the following one mark each questions: 14

- 1 Find $\Gamma\left(\frac{13}{2}\right)$
- 2 State relationship between beta and gamma functions.
- 3 Represent graphically the given saw-tooth function $f(x) = 2x, \quad 0 \leq x < 2$ and $f(x+2) = f(x)$ for all x .
- 4 For a periodic function f with fundamental period p , state the formula to find Laplace transform of f .
- 5 Find $L(e^{-3t}f(t))$, if $L(f(t)) = \frac{s}{(s-3)^2}$.
- 6 Find $L[(2t-1)^2]$.
- 7 Find the extension of the function $f(x) = x+1$, define over $(0,1]$ to $[-1,1] - \{0\}$ which is an odd function.
- 8 Is the function $f(x) = \begin{cases} x, & 0 \leq x \leq 2 \\ x^2, & 2 < x \leq 4 \end{cases}$; continuous on $[0,4]$? Give reason.
- 9 Is the differential equation $\frac{dy}{dx} = \frac{y}{x}$ exact? Give reason.
- 10 Give the differential equation of the orthogonal trajectory to the equation $y = cx^2$.
- 11 If $y = c_1y_1 + c_2y_2 = e^x(c_1 \cos x + c_2 \sin x)$ is a complementary function of a second order differential equation, find the Wronskian $W(y_1, y_2)$.
- 12 Solve $(D^2 + D + 1)y = 0$; where $D = \frac{d}{dt}$.
- 13 Is $u(t, x) = 50e^{(t-x)/2}$, a solution to $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} + u$?
- 14 Give an example of a first order partial differential equation of Clairaut's form.

Q.2 (a) Solve: $\frac{dy}{dx} = \frac{x^2 - x - y^2}{2xy}$. 03

- (b) Solve: $\frac{dy}{dx} + \frac{1}{x}y = x^3y^3$. 04
- (c) Find the series solution of $(x-2)\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} + 9y = 0$ about $x_0 = 0$. 07
- OR**
- (c) Explain regular-singular point of a second order differential equation and find the roots of the indicial equation to $x^2y'' + xy' - (2-x)y = 0$. 07
- Q.3** (a) Find the complete solution of $\frac{d^3y}{dx^3} + 8y = \cosh(2x)$. 03
- (b) Find solution of $\frac{d^2y}{dx^2} + 9y = \tan 3x$, using the method of variation of parameters. 04
- (c) Using separable variable technique find the acceptable general solution to the one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ and find the solution satisfying the conditions $u(0, t) = u(\pi, t) = 0$ for $t > 0$ and $u(x, 0) = \pi - x$, $0 < x < \pi$. 07
- OR**
- Q.3** (a) Solve completely, the differential equation $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \cos(2x) \sin x$. 03
- (b) Solve completely the differential equation $x^2 \frac{d^2y}{dx^2} - 6x \frac{dy}{dx} + 6y = x^{-3} \log x$. 04
- (c) (i) Form the partial differential equation for the equation $(x-a)(y-b) - z^2 = x^2 + y^2$. 07
- (ii) Find the general solution to the partial differential equation $xp + yq = x - y$.
- Q.4** (a) Find the Fourier cosine integral of $f(x) = \frac{\pi}{2}e^{-x}$, $x \geq 0$. 03
- (b) For the function $f(x) = \cos 2x$, find its Fourier sine series over $[0, \pi]$. 04
- (c) For the function $f(x) = \begin{cases} x; & 0 \leq x \leq 2 \\ 4-x; & 2 \leq x \leq 4 \end{cases}$, find its Fourier series. 07
- Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{16}$.
- OR**
- Q.4** (a) Find the Fourier cosine series of $f(x) = e^{-x}$, where $0 \leq x \leq \pi$. 03
- (b) Show that $\int_0^\infty \frac{\lambda^3 \sin \lambda x}{\lambda^4 + 4} d\lambda = \frac{\pi}{2} e^{-x} \cos x$, $x > 0$. 04
- (c) Is the function $f(x) = x + |x|$, $-\pi \leq x \leq \pi$ even or odd? Find its Fourier series over the interval mentioned. 07
- Q.5** (a) Find $L \left\{ \int_0^t e^u (u + \sin u) du \right\}$. 03
- (b) Find $L^{-1} \left\{ \frac{1}{s(s^2 - 3s + 3)} \right\}$. 04
- (c) Solve the initial value problem: $y'' - 2y' = e^t \sin t$, $y(0) = y'(0) = 0$, using Laplace transform. 07
- OR**
- Q.5** (a) Find $L\{t(\sin t - t \cos t)\}$. 03
- (b) Find $L^{-1} \left\{ \frac{e^{-2s}}{(s^2 + 2)(s^2 - 3)} \right\}$. 04
- (c) State the convolution theorem and verify it for $f(t) = t$ and $g(t) = e^{2t}$. 07
