Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III(New) • EXAMINATION – WINTER 2016

Subject Code:2130002 Date:30/12/2016

Subject Name:Advanced Engineering Mathematics

Time:10:30 AM to 01:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Answer the following one mark questions**	**14**
	1	Find $\Gamma\left(\frac{1}{2}\right)$.	
	2	State relation between beta and gamma function.	
	3	Define Heaviside's unit step function.	
	4	Define Laplace transform of f (t), t ≥ 0.	
	5	Find Laplace transform of $t^{\frac{-1}{2}}$.	
	6	Find L { $\frac{sinat}{t}$ }, given that L { $\frac{sint}{t}$ } = $tan^{-1}\{\frac{1}{s}\}$.	
	7	Find the continuous extension of the function f(x) = $\frac{x^2+x-2}{x^2-1}$ to x = 1	
	8	Is the function f(x) = $\frac{1}{x}$ continuous on [-1, 1] ? Give reason.	
	9	Solve $\frac{dy}{dx} = e^{3x-2y} + x^2e^{-2y}$.	
	10	Give the differential equation of the orthogonal trajectory of the family of circles $x^2 + y^2 = a^2$.	
	11	Find the Wronskian of the two function sin2x and cos2x .	
	12	Solve ($D^2 + 6D + 9$) x = 0; D = $\frac{d}{dt}$.	
	13	To solve heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ how many initial and boundary conditions are required.	
	14	Form the partial differential equations from z = f(x + at) + g(x –at).	
Q.2	(a)	Solve : (x+1)$\frac{dy}{dx} - y = e^{3x}(x+1)^2$.	03
	(b)	Solve : $\frac{dy}{dx} + \frac{ycosx + siny + y}{sinx + xcosy + x} = 0$	04
	(c)	Find the series solution of $\frac{d^2y}{dx^2} + xy = 0$.	07
		OR	
	(c)	Find the general solution of $2x^2y'' + xy' + (x^2 - 1)y = 0$ by using frobenius method.	07
Q.3	(a)	Solve : $(D^3 - 3D^2 + 9D - 27)$y = cos3x.	03
	(b)	Solve : $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x^2\sin(lnx)$.	04
	(c)	(i) Solve : $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2\partial y} = 2e^{2x}$.	03
		(ii) find the general solution to the partial differential equation	04

$(x^2 - y^2 - z^2)p + 2xyq = 2xz.$

OR

Q.3 **(a)** Solve : $(D^3 - D)y = x^3$. **03**

(b) Find the solution of $y'' - 3y' + 2y = e^x$, using the method of variation of parameters. **04**

(c) Solve $x\frac{\partial u}{\partial x}$ - 2y $\frac{\partial u}{\partial y} = 0$ using method of separation of variables. **07**

Q.4 **(a)** Find the Fourier cosine integral of $f(x) = e^{-kx}, x > 0, k > 0$ **03**

(b) Express f(x) = |x|, $-\pi < x < \pi$ as fouries series. **04**

(c) Find Fourier Series for the function f(x) given by **07**

$$f(x) = \begin{cases} 1 + \dfrac{2x}{\pi};\ -\pi \le x \le 0 \\ 1 - \dfrac{2x}{\pi}\ ;0 \le x \le \pi \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ \dots. = \frac{\pi^2}{8}$.

OR

Q.4 **(a)** Obtain the Fourier Series of periodic function function f(x) =2x, -1 < x < 1, p = 2L =2 **03**

(b) Show that $\int_0^\infty \frac{sin\lambda cos\lambda}{\lambda} d\lambda$ = 0, if x > 1. **04**

(c) Expand f(x) in Fourier series in the interval (0, 2π) if **07**

$$f(x) = \begin{cases} -\pi\ ;0 < x < \pi \\ x - \pi\ ;\pi < x < 2\pi \end{cases}$$

and hence show that $\sum_{r=0}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}$.

Q.5 **(a)** Find L$\{\int_0^t e^t \frac{sint}{t} dt\}$. **03**

(b) Find $L^{-1}\{\frac{2s^2-1}{(s^2+1)(s^2+4)}\}$. **04**

(c) Solve initial value problem : $y'' - 3y'$ + 2y = 4t + e^{3t} ,y(0) = 1 and $y'(0) = -1$,using Laplace transform. **07**

OR

Q.5 **(a)** Find L {tsin3tcos2t}. **03**

(b) Find $L^{-1}\{\frac{e^{-3s}}{s^2+8s+25}\}$. **04**

(c) State the convolution theorem and apply it to evaluate $L^{-1}\{\frac{s}{(s^2+a^2)^2}\}$. **07**

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