Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–III (NEW) EXAMINATION – WINTER 2017

Subject Code: 2130002 **Date:06/11/2017**

Subject Name: Advanced Engineering Mathematics

Time: 10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Solve the following differential equation using variable separable method **03**
$3e^x \tan y\, dx + (1+e^x)\sec^2 y\, dy = 0$

(b) Find the Laplace transform of $t\sin^2 3t$. **04**

(c) Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the Fourier expression of $f(x)$. **07**
Deduce that $\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + \ldots\ldots\ldots.$

Q.2 **(a)** Define rectangle function and saw-tooth wave function. Also sketch the graphs. **03**

(b) Find the general solution of the following differential equation : **04**
$\dfrac{d^3y}{dx^3} - 2\dfrac{dy}{dx} + 4y = e^x \cos x$

(c) Find the power series solution of $(1-x^2)y'' - 2xy' + 2y = 0$ about the ordinary point $x = 0$ **07**

OR

(c) Find the power series solution of $3x\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 0$ about the point $x = 0$, using Frobenius method. **07**

Q.3 **(a)** Express $f(x) = \begin{cases} 1, & for\ 0 \le x \le \pi \\ 0, & for\ x > \pi \end{cases}$ **03**
As a Fourier sine integral and hence evaluate $\displaystyle\int_0^\infty \frac{1-\cos(\pi\lambda)}{\lambda} \sin(x\lambda)\, d\lambda$

(b) Check whether the given differential equations is exact or not **04**
$(x^4 - 2xy^2 + y^4)\,dx - (2x^2 y - 4xy^3 + \sin y)\,dy = 0$
Hence find the general solution.

(c) Solve the following differential equation using the method of undetermined coefficient : $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 4y = 2x^2 + 3e^{-x}$ **07**

OR

Q.3 **(a)** Find the cosine series for $f(x) = \pi - x$ in the interval $0, < x < \pi$. **03**

(b) Solve the following differential equation $\dfrac{d^2y}{dx^2} + y = \sin x$ using the method of variation of parameters. **04**

(c) Solve the following Cauchy-Euler equation **07**
$x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + y = \log x . \sin(\log x)$

Q.4 **(a)** Find the orthogonal trajectory of the cardioids $r = a(1-\cos\theta)$ **03**

(b) Find the Laplace transforms of : **04**

(i) $e^{-3t}\,u(t-2)$

(ii) $\dfrac{1-\cos 2t}{t}$

(c) Solve the equation $\dfrac{\partial^2 z}{\partial x^2} - 2\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$ by the method of separation of variables. **07**

OR

Q.4 **(a)** Solve the following Bernoulli's equation: **03**

$$\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$$

(b) Find the inverse Laplace transforms of : **04**

(i) $\tan^{-1}\left(\dfrac{2}{s}\right)$

(ii) $\dfrac{s^3}{s^4 - a^4}$

(c) Find the complete solution of the following partial differential equations: **07**

(i) $\dfrac{\partial^3 z}{\partial x^3} - 3\dfrac{\partial^3 z}{\partial x^2 \partial y} + 4\dfrac{\partial^3 z}{\partial y^3} = e^{x+2y}$

(ii) $\dfrac{\partial^2 z}{\partial x^2} + 3\dfrac{\partial^2 z}{\partial x \partial y} + 2\dfrac{\partial^2 z}{\partial y^2} = x + y$

Q.5 **(a)** Form the partial differential equations by eliminating the arbitrary function from $f(x^2 + y^2,\, z - xy) = 0$ **03**

(b) Solve the following Lagrange's linear differential equation: **04**

$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

(c) Solve the following initial value problem using the method of Laplace transforms **07**

$y''' + 2y'' - y' - 2y = 0$ given that $y(0) = 1,\ y'(0) = 2,\ y''(0) = 2$

OR

Q.5 **(a)** Find the Laplace transform of the periodic function of the waveform **03**

$f(t) = \dfrac{2t}{3},\ \ 0 \le t \le 3\ ,\ \ f(t+3) = f(t)$

(b) Using the convolution theorem, find **04**

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\},\ a \neq b$$

(c) A tightly stretched string of length l with fixed ends is initially in equilibrium position. It is set vibrating by giving each point a velocity $v_0 \sin^3 \dfrac{\pi x}{l}$ .find the displacement $y(x,t)$. **07**

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