Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE - SEMESTER–III (New) EXAMINATION – WINTER 2018**

**Subject Code:2130002 Date:17/11/2018**

**Subject Name:Advanced Engineering Mathematics**

**Time:10:30 AM TO 01:30 PM Total Marks: 70**

**Instructions:**

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

| | | | MARKS |
|---|---|---|---|
| **Q.1** | **(a)** | Define: Dirac Delta function, Gamma function, Laplace Transform of a function. | **03** |
| | **(b)** | Solve $(D^2-4)y = 1+e^x$ ; where $D = d/dx$. | **04** |
| | **(c)** | Find the Fourier series for $f(x) = \begin{cases} \pi + x \ ; & -\pi < x < 0 \\ \pi - x \ ; & 0 < x < \pi \end{cases}$ | **07** |
| **Q.2** | **(a)** | Solve $\frac{d^4y}{dx^4} - 2\frac{d^2y}{dx^2} + y = 0$. | **03** |
| | **(b)** | Solve $Sinhx\ Cosy\ dx = Coshx\ Siny\ dy$. | **04** |
| | **(c)** | Find the Half range Cosine series for $f(x) = (x-1)^2$ in $(0,1)$. | **07** |
| | | **OR** | |
| | **(c)** | Show that current in a circuit containing Resistance $R$, Inductance $L$ and Constant emf $E$ is given by $i = \frac{E}{R}\left[1 - e^{-\frac{R}{L}t}\right]$. | **07** |
| **Q.3** | **(a)** | Solve $x^2y'' + xy' + y = 0$. | **03** |
| | **(b)** | Solve by the method of undetermined coefficients. $$y'' + 10y' + 25y = e^{-5x}$$ | **04** |
| | **(c)** | Solve using method of variation of parameters. $$y'' + 2y' + y = e^{-x}Cosx$$ | **07** |
| | | **OR** | |
| **Q.3** | **(a)** | State and prove First shifting theorem of Laplace Transform. | **03** |
| | **(b)** | Express $f(x) = \begin{cases} Sin\ x\ ; & 0 \ll x \ll \pi \\ 0 \ ; & x > \pi \end{cases}$ as Fourier Sine integral and evaluate $\int_0^\infty \frac{Sin\ \lambda x\ Sin\ \pi\lambda}{1-\lambda^2} d\lambda$ | **04** |
| | **(c)** | Solve in series the equation $y' = 3x^2y$. | **07** |
| **Q.4** | **(a)** | Find $L[t\ Sint]$ | **03** |
| | **(b)** | Find $L^{-1}\left[\frac{4s+5}{(s-1)^2(s+2)}\right]$ | **04** |
| | **(c)** | State Convolution theorem and hence find $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$ | **07** |
| | | **OR** | |
| **Q.4** | **(a)** | Define unit step function $u(t-a)$. Find $L[t^2u(t-2)]$. | **03** |
| | **(b)** | Solve the differential equation. $(D^3 - 2D^2 + 4D - 8)y = 0$ ; where $D = d/dx$ | **04** |

**(c)** Solve differential equation using Laplace transform. **07**

$$y'' + 2y' + y = e^{-t} \quad ; \quad y(0) = -1, y'(0) = 1$$

**Q.5** **(a)** Find Radius of convergence of the power series. **03**

$$\sum_{0}^{\infty} \frac{x^n}{n!}$$

**(b)** Solve the partial differential equation. **04**

$$p + q = z \;\; ; where\; p = \frac{\partial z}{\partial x} \;\; and\; q = \frac{\partial z}{\partial y}$$

**(c)** Prove that Laplace Equation in polar coordinates is **07**

$$\nabla^2 u = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

**OR**

**Q.5** **(a)** Form partial differential equation by eliminating arbitrary functions. **03**

$$f(xy + z^2, x + y + z) = 0$$

**(b)** Solve partial differential equation. **04**

$\frac{\partial^2 z}{\partial x^2} + z = 0$, given that when $x = 0; z = e^y$ and $\frac{\partial z}{\partial x} = 1$.

**(c)** Solve the partial differential equation using method of separation of variables. **07**

$$\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y} \quad ; \quad u(0, y) = 8e^{-3y}$$

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