Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2015

Subject Code: 2141905 **Date:26/05/2015**

Subject Name: Complex Variables & Numerical Methods

Time:10:30am-1.30pm **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 Do as Directed: 14

(1) Solve the equation $z^2+(2i-3)z+5-i=0$.

(2) Discuss the differentiability of $f(z)=x^2+iy^2$.

(3) Discuss the continuity of $f(z)=\begin{cases}\dfrac{\bar{z}}{z}; & z\neq 0\\ 0; & z=0\end{cases}$ at origin.

(4) Find the image of infinite strip $0\le x\le 1$ under the transformation $\omega = iz+1$. Sketch the region in $\omega -$plane.

(5) Find the second divided difference for the argument $x=1$, 2, 5 and 7 for the function $f(x)=x^2$.

(6) Evaluate $\oint_C \frac{e^z}{(z+i)}dz$, where $C:|z-1|=1$.

(7) Expand $f(z)=\frac{z-\sin z}{z^2}$ at $z=0$, classify the singular point $z=0$.

Q.2 **(a)** (1) Find the analytic function $f(z)=u+iv$, if $u-v=e^x(\cos y-\sin y)$. **04**

(2) If $f(z)=u+iv$ is analytic in domain D then prove that **03**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|\mathrm{Re}(f(z))|^2 = 2\,|f'(z)|^2.$$

(b) Using theory of residue, evaluate $\int_{-\infty}^{\infty}\frac{\cos x}{x^2+1}dx$. **07**

OR

(b) Expand $f(z)=\frac{1}{(z+1)(z-2)}$ in Laurent's series in the region (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$. **07**

Q.3 **(a)** (1)Evaluate $\int_C \bar{z}\,dz$ where C is along the sides of triangle having vertices $z=0,1,i$. **04**

(2) Determine bilinear transformation which maps the points $z=0,i,1$ into $\omega = i,-1,\infty$ **03**

(b) (1) Determine the points where $\omega=\left(z+\frac{1}{z}\right)$ is not conformal mapping. Also Find the image of circle $|z|=2$ under the transformation $\omega=\left(z+\frac{1}{z}\right)$. **04**

(2) Find the radius of convergence of $\sum_{n=1}^{\infty}\left(\frac{6n+1}{2n+5}\right)^2(z-2i)^n$. **03**

OR

Q.3 **(a)** (1) Using residue theorem, evaluate $\int_C \frac{e^z + z}{z^3 - z} dz$, where C: $|z| = \frac{\pi}{2}$. **04**

(2) State Cauchy integral theorem. Evaluate $\int_C \left(\frac{3}{z-i} - \frac{6}{(z-i)^2} \right) dz$, where C : $|z| = 2$. **03**

(b) (1) If $x + \frac{1}{x} = 2\cos\theta$, prove that (i) $x^n + \frac{1}{x^n} = 2\cos n\theta$ and **04**

(ii) $\frac{x^{2n}+1}{x^{2n-1}+x} = \frac{\cos n\theta}{\cos(n-1)\theta}$

(2) Determine and sketch the image of region $0 \le x \le 1, \quad 0 \le y \le \pi$ under the transformation $\omega = e^z$ **03**

Q.4 **(a)** (1) Construct Newton's forward interpolation polynomial for the following data: **04**

x:	4	6	8	10
y:	1	3	8	16

Hence evaluate y for x=5.

(2) Evaluate $\int_0^1 e^{-x^2} dx$ by using Gaussian Quadrature formula with n=3. **03**

(b) Using the power method, find the largest eigen value of the $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. **07**

OR

Q.4 **(a)** (1) Find the real root of the equation $x \log_{10}{}^{x} = 1.2$ by Regula false method **04**

(2) A river is 80 meters wide. The depth 'd' in meters at a distance x meters from one bank is given by the following table. Calculate the area of cross section of the river using Simpson's $\frac{1}{3}$ rd rule. **03**

x:	0	10	20	30	40	50	60	70	80
y:	0	4	7	9	12	15	14	8	7

(b) Use Lagrange's method to find polynomial of degree three for the data **07**

x:	-1	0	1	3
y:	2	1	0	-1

Hence find the value of $x = 2$.

Q.5 **(a)** State diagonal dominant property. Using Gauss-Seidel method to solve **07**
$6x + y + z = 105,\ 4x + 8y + 3z = 155,\ 5x + 4y - 10z = 65$

(b) Using the Runge- Kutta method of fourth order, Solve $10\frac{dy}{dx} = x^2 + y^2,\ y(0) = 1$ at $x = 0.2$ and $x = 0.4$ taking h=0.1 **07**

OR

Q.5 **(a)** Derive Euler's formula for initial value problem $\frac{dy}{dx} = f(x,y); y(x_0) = y_0$. Hence, use it find the value of y for $\frac{dy}{dx} = x + y; y(0) = 1$ when $x = 0.1, 0.2$ with step size h=0.05. Also Compare with analytic solution. **07**

(b) (1)Use Gauss elimination method to solve the equation **04**
$x + 4y - z = -5, x + y - 6z = -12, 3x - y - z = 4.$

(2) Use Newton- Raphson method, derive the iteration formula for $\sqrt{N}$ .Also find $\sqrt{28}.$ **03**

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