Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(New) EXAMINATION – SUMMER 2016

Subject Code:2141905 **Date:26/05/2016**

Subject Name:Complex Variables and Numerical Methods

Time:10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Short Questions**	
	1	Express $\sqrt{3}-i$ into polar form.	**1**
	2	Evaluate $\Delta \cos x$.	**1**
	3	Evaluate $\lim\limits_{z \to i} \dfrac{z-i}{z^2+1}$	**1**
	4	Find the radius of convergence for the series $\sum_{n=0}^{\infty} z^n$	**1**
	5	Write formula for Simpson's 3/8 rule.	**1**
	6	Find the fixed points of $w = \dfrac{z-1}{z+1}$	**1**
	7	Give the names of two iterative methods for the solution of system of linear equations.	**1**
	8	State the theorem, " Cauchy's Integral Formula".	**1**
	9	Find the pole and its order for $f(z) = \dfrac{e^z - 1}{z^3}$	**1**
	10	Find the third divided difference with arguments 2, 4, 9, 10 of the function $f(x) = x^3 - 2x$.	**1**
	11	Find Res $(f(z),1)$ for $f(z) = \dfrac{1}{z(z-1)}$	**1**
	12	Find the interval for $x^3 - x - 11 = 0$ in which the root lies.	**1**
	13	State DeMoivre's Theorem.	**1**
	14	Write iterative formula to find $\sqrt{7}$ using Newton-Raphson method.	**1**
Q.2	**(a)**	Find all the values of $\left(\dfrac{1}{2}+\dfrac{\sqrt{3}}{2}i\right)^{3/4}$.	**03**
	(b)	Show that the function $f(z) = xy + iy$ is continuous everywhere but is not analytic.	**04**
	(c)	Attempt the following	
	(i)	If $u = e^x(x\cos y - y\sin y)$, find the analytic function *f(z)*.	**3**
	(ii)	Find the value of $\int_0^{2+i} (\bar{z})^2 \, dz$, along the real axis from 0 to 2 and then vertically from 2 to 2 + *i*.	**4**
		OR	
	(c)	Attempt the following	
	(i)	If *f(z)* is a regular function of *z*, prove that	**3**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

(ii) Prove that $\sinh^{-1} x = \log\left\{x+\sqrt{x^2+1}\right\}$ **4**

Q.3 (a) Evaluate $\int_C \frac{e^{2z}}{(z+1)^3}dz,$ where C: $4x^2+9y^2=16$ using residue theorem. **03**

(b) Find the bilinear transformation which transforms $z = 2, 1, 0$ into $w= 1, 0, i$. **04**

(c) Expand $\frac{1}{z(z^2-3z+2)}$ about $z = 0$, for the regions $(i) 0<|z|<1$ $(ii) 1<|z|<2$ $(iii) |z|>2.$ **07**

OR

Q.3 (a) Evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)}dz,$ where C is the circle $|z-i|=2.$ **03**

(b) Find the image of $|z-3i|=3$ under the mapping $w=\frac{1}{z}.$ **04**

(c) Evaluate $P.V.\int_{-\infty}^{\infty}\frac{x\cos x}{x^2+9}dx.$ **07**

Q.4 (a) Using Newton's divided difference formula, find a polynomial function satisfying the following data: **03**

x	-4	-1	0	2	5
f(x)	1245	33	5	9	1335

(b) The table below gives the values of function *y=tanx*. Obtain the value of *tan(0.40)* using Newton's backward interpolation. **04**

x	0.10	0.15	0.20	0.25	0.30
y=tanx	0.1003	0.1511	0.2027	0.2553	0.3093

(c) Use the power method to find the largest eigen value and corresponding eigen vector of the matrix $A=\begin{bmatrix}1&6&1\\1&2&0\\0&0&3\end{bmatrix}.$ **07**

OR

Q.4 (a) Find the root of the equation $x^2-4x-10=0$ correct to three decimal places by using bisection method. **03**

(b) Compute f(4) from the tabular value given **04**

x	2	3	5	7
f(x)	0.1506	0.3001	0.4517	0.6259

Using Lagrange interpolating polynomial.

(c) Solve the following system of equations by Gauss – Jordan method: $10x+y+z=12, 2x+10y+z=13, x+y+5z=7.$ **07**

Q.5 (a) The velocity *v* of a particle at distance *s* from point on its path is given by the following table: **03**

s (meter)	0	10	20	30	40	50	60
v (meter/Sec)	47	58	64	65	61	52	38

Find the time taken to travel 60 meter, using Simpson's 1/3 rule. (Use $v=\frac{ds}{dt}$).

(b) Use the method of Regula Falsi to find the root of $x=e^{-x}$ correct to three decimal places. **04**

(c) Use fourth order Runge Kutta method to find the value of *y* at *x = 1* **07**

given that $y' = \frac{y-x}{y+x}$ such that $y(0) = 1$.(Take $h = 0.5$)

OR

Q.5 **(a)** Use Gauss Seidel method to solve: **03**
$83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$, $3x + 8y + 29z = 71$.

(b) Evaluate the integral $\int_{-2}^{6} (1+x^2)^{3/2} dx$ by the Gaussian formula for $n = 3$. **04**

(c) Using Euler's method solve for y at $x = 0.1$ from $\frac{dy}{dx} = x + y + xy$, $y(0) = 1$, in five steps. **07**

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