Seat No.: _____ **Enrolment No._____**

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) - EXAMINATION – SUMMER 2017

Subject Code: 2141905 **Date: 30/05/2017**

Subject Name: Complex Variables and Numerical Methods

Time: 10:30 AM to 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Short Questions**	**14**
	1	Write the formula of $cosh(x+y)$	
	2	Write the general value of $Log(1+i)$	
	3	Does $\lim_{z\to 0} \frac{z}{\lvert z\rvert}$ exist ?	
	4	State Liouville's Theorem for Complex analysis.	
	5	State Maximum Modulus Principle for Complex analysis.	
	6	Find the radius of convergence of $\sum_{n=1}^{\infty} (6+8i)^n z^n$	
	7	Write the zeroes and poles of $\frac{3z+1}{z(2z-5)^2}$ with their order.	
	8	Write the condition for being a harmonic function.	
	9	Prove that $\Delta = E - 1$	
	10	Prove that $\delta = E^{1/2} - E^{-1/2}$	
	11	Write the formula to estimate the error in calculating the integral using Simpson's 3/8th rule.	
	12	What is the rate of convergence of secant method.	
	13	Write the formula of 2nd order Runge-Kutta method to solve a first order differential equation.	
	14	Write the names of two numerical methods to find an eigen value of a matrix.	
Q.2	**(a)**	Simplify $\left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^8$	**07**
	(b)	Show that the function $f(z)=\sqrt{\lvert xy\rvert}$ satisfies the Cauchy-Riemann equations at the origin but $f'(0)$ fails to exist.	**07**
		OR	
	(b)	Find the analytic function $f(z)$ if $u-v=\frac{e^y-\cos x+\sin x}{\cosh y-\cos x}$ with $f\left(\frac{\pi}{2}\right)=\frac{3-i}{2}$.	**07**
Q.3	**(a)**	Evaluate $\int_0^{2+i}(\bar{z})^2\,dz$ along the shortest path joining the end points.	**05**
	(b)	Determine the Taylor's series expansion of $\frac{2z^2+9z+5}{(z-3)(z+2)^2}$ with center at $z_0=1$	**05**
	(c)	Evaluate $\int_0^{\infty}\frac{x^2dx}{(x^2+1)(x^2+4)}$	**04**

OR

Q.3 **(a)** Evaluate $\oint_C \frac{\cos z}{(z-\frac{\pi}{4})^3} dz$ where C is the circle $\left|z-\frac{\pi}{2}\right| = 1$ using Cauchy's Integral formula **05**

(b) Evaluate $\oint_C \frac{z^3 - z^2 + z - 1}{z^3 + 4z} dz$ where C is the circle $|z| = 3$ using Cauchy's residue theorem **05**

(c) Evaluate $\int_0^{2\pi} \frac{d\theta}{2 + \cos\theta}$ **04**

Q.4 **(a)** The following table gives the population of a town during the six censuses. Estimate the increase in the population during the period from 1986 to 1988. **05**

Year	1941	1951	1961	1971	1981	1991
Population in Thousands	12	15	20	27	39	52

(b) Evaluate $\int_0^6 \frac{dx}{1+x}$ using Simpson's 1/3 rd rule. Hence obtain the value of ln 7 **05**

(c) Find the Mobius Transform which sends the points z = 1, i, - 1 into the points w = 2, i, - 2 **04**

OR

Q.4 **(a)** Find the Lagrange's interpolation polynomial from the following **05**

x	0	1	4	5
f(x)	1	3	24	39

(b) The speed *v* of a car, after it starts is shown in the following table. *v* is in *mt/sec* and *t* is in seconds. **05**

t	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.08	18.90	21.60	18.54	10.26	4.5	4.5	5.4	9

Using Simpson's one-third rule, find the distance travelled by the car in 2 minutes.

(c) Determine the region of w – plane into which the region bounded by $x = 1$, $y = 1$ and $x + y = 1$ is mapped by the transformation $w = z^2$ **04**

Q.5 **(a)** Find a real root of $2x - \log x = 7$ correct up to four decimal places using Newton–Raphson method. **05**

(b) Solve $\frac{dy}{dx} + 2xy^2 = 0$ with initial condition $y(0) = 1$ for $x = 1$ in five steps using Modified Euler's Method. **05**

(c) Solve the linear system of equations $x + 17y - 2z = 48$, $2x + 2y + 18z = 30$, $30x - 2y + 3z = 75$ correct up to four decimal places using Gauss-Jacobi method. **04**

OR

Q.5 **(a)** Find a real root of $3x = \sqrt{1 + \sin x}$ correct up to four decimal places using false position method. **05**

(b) Solve $\frac{dy}{dx} = x + y^2$ with initial condition $y(0) = 1$ for $x = 0.2$ in two steps using 4th order Runge-Kutta Method. **05**

(c) Use power method to find the dominant eigen value of $\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$ **04**

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