

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-IV (NEW) - EXAMINATION – SUMMER 2018

Subject Code:2141905

Date:17/05/2018

Subject Name:Complex Variables and Numerical Methods

Time:10:30 AM to 01:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** Separate real and imaginary parts of $f(z) = e^z$. Also prove that it is analytic **03**
everywhere.
- (b)** Write De Moivre's theorem. Find out cube root of i . **04**
- (c)** Apply Gauss elimination method to solve system of equations: **07**
 $6x + 2y + 8z = 26$
 $3x + 5y + 2z = 8$
 $8y + 2z = -7$

- Q.2 (a)** Evaluate $\oint_C \frac{e^{2z}}{(z - \ln 1.5)^3} dz$, where C is the square with vertices $(1,0)$, $(0,1)$, **03**
 $(0,-1)$ and $(-1,0)$
- (b)** Verify whether $u(x, y) = x^2 - y^2$ is harmonic or not? Also find out its **04**
harmonic conjugate.
- (c)** State Cauchy – Riemann theorem. Write C-R equations in polar form and **07**
verify it for $f(z) = \frac{z}{\bar{z}}$ in polar form.

OR

- (c)** Evaluate $\int_C |z|^2 dz$ where C is the boundary of the square with vertices $(0,0)$, **07**
 $(1,0)$, $(1,1)$, $(0,1)$.

- Q.3 (a)** Find the radius of convergence of the series $\sum_{n=2}^{\infty} \frac{z^n}{n(\log n)^3}$ **03**

- (b)** Evaluate $\oint_C \frac{z^2 + 1}{(z^2 - 1)} dz$, if C is the circle of unit radius with center (I) at $z = 1$ **04**
and (II) at $z = -1$.

- (c)** Evaluate following integrals using residue: **07**

(1) $\oint_C \frac{2z^2 + 3}{z(z+1)(z+2)} dz$, where $C: |z| = 1.6$

(2) $\int_{-\infty}^{\infty} \frac{3x+2}{x(x-4)(x^2+9)} dx$

OR

- Q.3 (a)** Find series expression for $f(z) = \tan^{-1} z$ at $z = 0$ **03**
- (b)** Find the bilinear transformation which maps the points $z = 1, i, -1$ in to the points $w = i, 0, 1$ **04**
- (c)** Find all Taylor and Laurent series of $f(z) = \frac{-2z+3}{z^2-3z+2}$ with center 0. **07**

- Q.4 (a)** Evaluate $\int_3^8 x^2 \log x dx$ using Trapezoidal rule with step size $h = 1$ **03**
- (b)** Apply Lagrange's interpolation formula to evaluate $Y(3)$ using data given below: **04**

X	0	1	2	4
Y	-1	2	7	23

- (c)** Find the 3th root of the 119 correct up to five decimal places, using Newton Raphson method. **07**

OR

- Q.4 (a)** Evaluate $\int_0^1 \frac{dt}{1+t}$ using Two point Gaussian formula. **03**
- (b)** Use Bolzano method to find the positive root of $x = \cos x$ correct up to three decimal places. Use initial values 0.72 and 0.75 **04**
- (c)** Use appropriate Newton's formula to find the values of $Y(21)$ and $Y(28)$ from the data given below: **07**

X	20	23	26	29
Y	79.8	121.44	175.5	243.6

- Q.5 (a)** Apply Gauss Jacobi method to solve system of linear equation as under: **07**
- $20x + 2y + z = 30$
 $x - 40y - 3z = -75$
 $2x - y + 10z = 30$
- (b)** Given that $\frac{dy}{dx} = 3x + y$ with $y(1) = 1.3$, consider step size $h = 0.1$. Find **07**
- $y(1.1)$ and $y(1.2)$ using Euler's method. Also find $y(1.3)$ using Runge- Kutta method of order 4.

OR

- Q.5 (a)** Determine the largest Eigen value and the corresponding eigen vector of the matrix $A = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 1 \\ 2 & 1 & 8 \end{bmatrix}$ **07**
- (b)** Using Taylor's Series method solve $\frac{dy}{dx} = x^2 y - 1, y(0) = 1$. Also find **07**
- $y(0.03)$
