Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(NEW) – EXAMINATION – SUMMER 2019

Subject Code:2141905 Date:09/05/2019

Subject Name: Complex Variables and Numerical Methods

Time:02:30 PM TO 05:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** State De'Movier's. Find arg [i / (- 2 – 2i)] . **03**

(b) Define the operators Δ, ∇ and E. Prove that $E\nabla = \Delta$. **04**

(c) State Cauchy – Riemann Equations. Show that **07**
(i) f(z) = sin z is everywhere analytic
(ii) f(z) = xy + iy is nowhere analytic.

Q.2 **(a)** State the formula for $\sin^{-1} z$. Find $\sin^{-1}(-i)$ **03**

(b) Find analytic function f(z) = u + iv, if u = 2x(1 – y). **04**

(c) Classify the singularities of the analytic function. **07**
In each of the following case, identify the singular point and its type with justification.
(i) $z^2/(z+1)$ (ii) $\sin z/z$ (iii) $(1-\cosh z)/z^3$

OR

(c) Use residues to evaluate the improper integral: **07**

$$\int_0^{\infty} \frac{x^2\,dx}{(x^2+1)(x^2+4)}$$

Q.3 **(a)** Evaluate the integral $\int_C \bar{z}\, dz$, when C is the right-hand half $z = 2e^{i\theta}\left(-\pi/2 \le \theta \le \pi/2\right)$, of the circle |z| = 2 from – 2i to 2i. **03**

(b) Show that the mapping by w = 1/z transforms circles and lines into circles and lines. **04**

(c) Give two Laurent series expansions in powers of z for the function $f(z) = 1/[z^2(1-z)]$ and specify the regions in which those expansions are valid. **07**

OR

Q.3 **(a)** Find the bilinear transformation which transforms $z_1 = \infty$, $z_2 = i$, $z_3 = 0$ into $w_1 = 0$, $w_2 = i$, $w_3 = \infty$. **03**

(b) Determine and sketch the region : **04**
(i) $0 \le \arg z \le \pi/4$,(ii) | 2z + 3| > 4,
Which of them are domains?

(c) State the Cauchy's Integral Formula and its extension. Hence evaluate integral $\int_C \frac{z+4}{z^2-2z+5}\,dz$, where C is circle |z +1 + i | =2. **07**

Q.4	**(a)**	Evaluate $\int_3^7 x^2 \log x\, dx$ taking four sub-intervals by trapezoidal rule.	**03**
	(b)	Apply Bisetion method to find a real root of the equation $2x^3 - 5x + 1 = 0$ correct to 2 decimal places.	**04**
	(c)	Given f(1)=22, f(2)=30, f(4)=82, f(7)=106, f(12)=206, find f(8) using Lagrange's interpolation formula.	**07**

OR

Q.4 **(a)** The velocity of a car (running on a straight road) at intervals of 2 minutes are given below. **03**

Time (in min.):	0	2	4	6	8	10	12
Velocity (in km/hr):	0	22	30	27	18	7	0

Apply Simpson's $1/3^{rd}$ rule to find the distance covered by the car.

(b) Newton Raphson method find a root of the equation xsinx +cosx = 0 correct to four decimal places (taking initial guess $x_0 = \pi$). **04**

(c) State Striling's Interpolation formula. **07**
Interpolate by means of Gauss' forward formula the population for the year 1936 given the following data:

Year:	1901	1911	1921	1931	1941	1951
Population (1000s)	12	15	20	27	39	52

Q.5	**(a)**	Using secant method find a real root of the equation $x^3 - 5x + 1 = 0$ up to three iterations.	**03**
	(b)	Use Runge-Kutta method to solve $y' = xy$, $y(0)=1$, for $x = 0.2$, with $h=0.1$.	**04**
	(c)	Using Gauss-Seidel method to solve the following system correct to 3 decimal places: $83x + 11y - 4z = 95$, $7x + 52y + 13z = 104$, $3x + 8y + 29z = 71$.	**07**

OR

Q.5	**(a)**	Solve $x \log_{10} x = 1.2$ by Regula Falsi method correct to two decimal places.	**03**
	(b)	Solve $y' = 1 - y$, $y(0) = 0$ in [0, 0.3] by modified Euler's method taking $h = 0.1$.	**04**
	(c)	Find the largest eigenvalue and corresponding eigenvector using power method, for	**07**

$$A = \begin{bmatrix} 4 & 4 & 2 \\ 4 & 4 & 1 \\ 2 & 1 & 8 \end{bmatrix}, \text{ taking } X_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

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