

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-IV (New) • EXAMINATION – WINTER 2016

Subject Code: 2141905

Date: 18/11/2016

Subject Name: Complex Variables and Numerical Methods

Time: 02:30 PM to 05:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

		MARKS
Q.1	Short Questions	14
1	Find the principle angle of $\frac{1}{\sqrt{3}+i}$.	
2	Define analytic function.	
3	Write necessary condition for differentiability of $f(z)$.	
4	Find the real part of $f(z) = \frac{1}{z-2i}$ at point $1+i$.	
5	Define harmonic function.	
6	Separate real and imaginary part of $f(z) = z^2$.	
7	Classify the singular point $z=0$ for the function $f(z) = \frac{1}{z^4 - 4z^2}$.	
8	Show that $\Delta = E - 1$.	
9	Check whether $w = \frac{1}{z}$ is conformal mapping or not.	
10	Write trapezoidal rule.	
11	When does Newton- Raphson method fail to find root of equation.	
12	Can we use Gauss- Seidel method to solve the system of linear equation $3x + y + 2z = 8, 2x + 3y + z = 9, x + 2y + 3z = 6$.	
13	Express $y = x^2$ in factorial notation.	
14	Find $[a, b]$ for $f(x) = \frac{1}{x}$.	
Q.2	(a) Check whether functions $f(z) = z^{\frac{3}{2}}$ is analytic or not. Also find derivative of $f(z)$.	03
	(b) Find the bilinear transformation which maps $z = 1, i, -1$ into the points $w = 0, 1, \infty$.	04
	(c) Evaluate the Cauchy's principle value of $\int_{-\infty}^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 9)}$.	07
	OR	
	(c) Show that $u(x, y) = e^x \cos y$ is harmonic function. Also find harmonic conjugate of $u(x, y)$.	07
Q.3	(a) Using Cauchy integral formula, Evaluate $\int_C \frac{3z^2 + 2}{(z-2)(z^2 + 4)} dz$ where C is a circle $ z-2 = 2$.	03

- (b)

Find and plot all the roots of $\sqrt[3]{8i}$.

04
- (c)

Evaluate $\int_C z^2 dz$ where C is taken along triangle in z-plane having vertices $z = \pm i, z = -1$ taken in counter clockwise sence.

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OR

- Q.3 (a)

Expend $f(z) = \frac{e^z}{(z-1)^2}$ about $z=1$. Also classify singular point $z=1$.

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- (b)

Discuss the continuity of $f(z) = \begin{cases} \bar{z}^2 & z \neq 0 \\ z & z = 0 \end{cases}$ at $z=0$.

04
- (c)

Expand $f(z) = \frac{1}{(z+2)(z+4)}$ valid for the region
(i) $|z| < 2$ (ii) $2 < |z| < 4$ (iii) $|z| > 4$.

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- Q.4 (a)

Solve by Gauss Elimination method
 $x + 2y + z = 3; 2x + 3y + 3z = 10; 3x - y + 2z = 13$.

03
- (b)

Derive iterative formula to find $\sqrt{N}$. Use this formula to find $\sqrt{28}$.

04
- (c)

Determine the polynomial by Newton's forward difference formula from the following table:

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x:	0	1	2	3	4
y:	-10	-8	-8	-4	40

Also find y when $x=1.5$.

OR

- Q.4 (a)

Using Euler's method, find an approximate value of corresponding to $x=0.3$, given that $\frac{dy}{dx} = \frac{y-x}{y+x}; y(0) = 1$. Take $h = 0.1$.

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- (b)

Find a real root of the equation $x^3 - 2x - 5 = 0$ using secant method correct to three decimal places taking $x_0 = 2$ and $x_1 = 3$.

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- (c)

Determine the interpolating polynomial of degree three using Lagrange's interpolation for the table below:

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x:	-1	0	1	3
y:	2	1	0	-1

Also find the value of y when $x=2$.

- Q.5 (a)

The velocity v of a particle at a distant s from a point on its path is given by the following table:

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s (ft)	0	10	20	30	40	50	60
v(ft/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft. using Simpson's $\frac{3}{8}$ th rule.

- (b)

Using Runge- Kutta method of fourth order to calculate $y(0.2)$ given that $\frac{dy}{dx} = x + y; y(0) = 1$, taking $h=0.1$.

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- (c)

Solve by Gauss-Seidel method
 $10x + y + z = 6; x + 10y + z = 6; x + y + 10z = 6$.

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OR

- Q.5 (a)

Evaluate $\int_0^1 \frac{1}{1+x} dx$ by Gaussian formula with two point.

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- (b)

Find a positive root of $xe^x - 2 = 0$ by the method of False position.

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- (c) Find the dominant eigen value of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by power method and hence find the other eigen value also. 07
