Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2017

Subject Code: 2141905 **Date: 09/11/2017**

Subject Name: Complex Variables and Numerical Methods

Time: 02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1	**(a)**	Find the fourth root of -1	**03**
	(b)	Prove that $(i)E = 1+\Delta \quad (ii)\nabla = 1-E^{-1}$	**04**
	(c)	Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$. Also find the image of $\lvert z\rvert < 1$	**07**
Q.2	**(a)**	Prove that $\sinh^{-1} x = \log\left\{x + \sqrt{x^2+1}\right\}$	**03**
	(b)	Show that the function $u = x^3 - 3xy^2$ is harmonic and find the corresponding analytic function	**04**
	(c)	If $f(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\lvert Rf(z)\rvert^2 = 2\lvert f'(z)\rvert^2$	**07**
		OR	
	(c)	Using the C-R equations, show that $f(z) = z^3$ is analytic everywhere	**07**
Q.3	**(a)**	Evaluate $\oint_C \frac{e^z}{z^2+1}dz$ over the circle $\lvert z\rvert = 2$	**03**
	(b)	Find the image of $\lvert z-3i\rvert = 3$ under the mapping $w = \frac{1}{z}$	**04**
	(c)	Find the Laurent series expansion of $f(z) = \frac{1}{6-z-z^2}$ in (i) the domain $\lvert z\rvert < 2$ (ii) the domain $2 < \lvert z\rvert < 3$ (iii) $\lvert z\rvert > 3$	**07**
		OR	
Q.3	**(a)**	Evaluate $\int_C \{(x+y)dx + x^2 ydy\}$ along $y = x^2$ having (0,0), (3,9) end points	**03**
	(b)	Find the radii of convergence of $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{n^2} z^n$	**04**
	(c)	Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and residue at each pole. Hence evaluate $\oint_C f(z)dz$ where C is the circle $\lvert z\rvert = 3$	**07**
Q.4	**(a)**	Find $\int_{10}^{16} ydx$ by Simpson's 1/3 rule from the following data:	**03**

x	*10*	*11*	*12*	*13*	*14*	*15*	*16*
y	*1.02*	*0.94*	*0.89*	*0.79*	*0.71*	*0.62*	*0.55*

(b) Find *y(300)* from the following data: **04**

x	*50*	*100*	*150*	*200*	*250*
y	*618*	*724*	*805*	*906*	*1032*

(c) Use second order Runge-Kutta method to solve $\frac{dy}{dx} = x + \sqrt{y}$, given *y(0)=4* at *x=0.2* by taking *h=0.1* **07**

OR

Q.4 **(a)** Find a real root of $x^3 - 5x + 3 = 0$, correct to three decimal places, using Newton-Raphson method **03**

(b) Use Lagrange interpolation to find *f(4)* from the following data: **04**

x	*2*	*3*	*5*	*7*
f(x)	*0.1506*	*0.3001*	*0.4517*	*0.6259*

(c) Using Euler's method solve for *y* at *x=0.1* from $\frac{dy}{dx} = x + y + xy$, y(0)=1 in five steps **07**

Q.5 **(a)** Use the method of Regula Falsi to find the root of $x = e^{-x}$ correct to three decimal places **03**

(b) Evaluate $\int_0^3 \frac{dx}{1+x}$ with *n=6* by using Simpson's *3/8* rule **04**

(c) Using Gauss-Seidel method solve: **07**
$3x - 0.1y - 0.2z = 7.85, 0.1x + 7y - 0.3z = -19.3, 0.3x - 0.2y + 10z = 71.4$

OR

Q.5 **(a)** Evaluate $\int_0^1 e^{-x^2} dx$ by using the Gaussian integration formula for *n = 3* **03**

(b) Let *f(40)=836, f(50)=682, f(60)=436, f(70)=272*. Use Stirling's formula to find *f(55)* **04**

(c) Use power method to find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$ **07**

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