Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV (NEW) EXAMINATION – WINTER 2018

Subject Code:2141905 Date:22/11/2018

Subject Name:Complex Variables and Numerical Methods

Time: 02:30 PM TO 05:30 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find the roots of the equation $z^2 + 2iz + (2 - 4i) = 0$ **03**

(b) Show that $f(z) = x^2 + iy^2$ is differentiable only at $z = 0$ and $f'(0) = 0$. **04**

(c) Solve the following system of equation by Gauss-Seidal method correct to three decimal places. **07**

$$\begin{aligned} 4x + y + 2z &= 11 \\ x + 5y - z &= 8 \\ 6x + 2y + 8z &= 15 \end{aligned}$$

Q.2 **(a)** Evaluate $\int_{0}^{2+i} z^2\,dz$ along the line joining the points $(0,0)$ and $(2,1)$. **03**

(b) Determine the mobius transformation that maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively. **04**

(c) Prove that the n^{th} roots of unity are in geometric progression. Also show that their sum is zero. **07**

OR

(c) Verify that C-R equation are satisfied at $z = 0$ for the function $f(z) = \begin{cases} \frac{x^3}{x} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$ **07**

Q.3 **(a)** Evaluate $\oint_C \begin{vmatrix} z & 0 \\ 0 & z^2 \end{vmatrix} dz$, where $C: |z| = 2$. **03**

(b) Find the radius of convergence of **04**

$$\sum_{n=1}^{\infty} \left(\frac{6n+1}{2n+5}\right)^n (z-2i)^n$$

(c) Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{5 - 3\sin\theta}$ **07**

OR

Q.3 **(a)** Expand $f(z) = \frac{z^2 - 1}{z+1}$ as a Taylor's series about the point $z = 0$. **03**

(b) Check whether $f(z) = \sin z$ is analytic or not. If analytic find its derivative. **04**

(c) Evaluate $\oint_C \frac{z^2 - 2z + 1}{z^2 + 4z}\,dz$ counter clockwise around C, where C is $\|z\| = 1$ and $\|z\| = 3$. **07**

Q.4 **(a)** Using Newton's forward formula , find the value of $f(1.6)$ if **03**

x	1	1.4	1.8	2.2
f(x)	3.49	4.82	5.96	6.5

(b) Find the Lagrange interpolating polynomial from the following data **04**

x	0	1	4	5
f(x)	1	3	24	39

(c) Find a root of $x^4 - x^3 - 10x+7 = 0$ correct to three decimal places between $a = -2$ and $b = -1$ by Newton-Raphson method. **07**

OR

Q.4 **(a)** Solve the system of equation by Gauss elimination method. **03**

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

(b) Compute $f(8)$ from the following values using Newton's Divided difference formula **04**

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

(c) Evaluate $\int_0^{\pi} \frac{1}{1+\sin x}\,dx$, taking $h = 1$ and using Simpson's $\frac{1}{3}$ rule. Hence obtain approximate value of $\log_e 7$. **07**

Q.5 **(a)** Evaluate $A^m e^{Ax}$ **03**

(b) Use power method to find the largest of Eigen values of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$ **04**

(c) Use Euler's method to obtain an approximate value of $y(0.4)$ for the differential equation $y' = x + y, y(0) = 1$ with $h = 0.1$. **07**

OR

Q.5 **(a)** Prove that $\Delta(\log f(x)) = \log(1 + \Delta)$ **03**

(b) Evaluate $I = \int_{-1}^{1} \frac{dx}{1+x^2}$ by one point, two point and three point Gaussian formula. **04**

(c) Determine $y(0.1), y(0.2)$ correct upto four decimal places by fourth order Runge-Kutta method from $\frac{dy}{dx} = 2x + y, y(0) = 1$ **07**

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