Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– 4 (NEW SYLLABUS) EXAMINATION- SUMMER 2016

Subject Code:2141005 Date: 08/06/2016

Subject Name: SIGNALS AND SYSTEMS

Time:10:30 AM to 01:00 PM Total Marks: 70

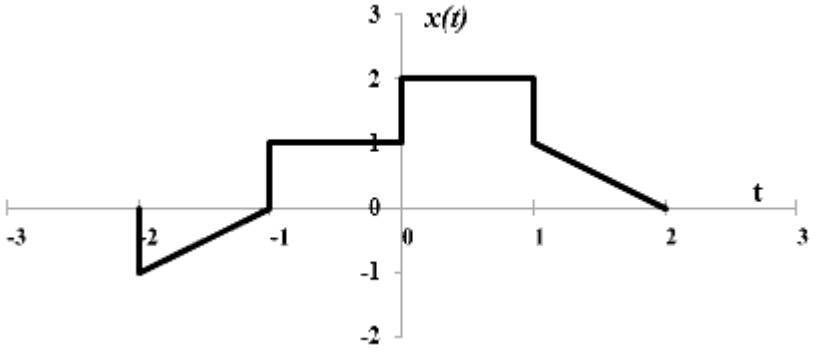
Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

			MARKS
Q.1		**Short Questions**	**14**
	1	Sketch the waveform of the following signal: $x(t) = u(t+1) - 2u(t) + u(t-1)$	
	2	If the response of LTI continuous time system to unit step signal is $\frac{1}{2} - \frac{1}{2}e^{-2t}$, then find the impulse response of the system.	
	3	Find inverse Laplace transform of $\frac{1}{s}(1 - e^{-2s})$	
	4	Fill in the blanks: "An energy signal has _____ average power, whereas a power signal has _____ energy."	
	5	Find the output of an LTI system with impulse response $h(t) = \delta(t-3)$ for the input $x(t) = \cos 4t + \sin \pi t$.	
	6	Evaluate the following integrals. **(i)** $\int_1^2 (3t^2+1)\delta(t)dt$ **(ii)** $\int_0^\infty t^2\delta(t-6)dt$	
	7	Determine whether following signal is periodic or not. If it is periodic, find fundamental period. $x[n] = (-1)^n$	
	8	Determine z- transform and its ROC of $x[n] = u[n]$.	
	9	Write Fourier Transform of $x(t) = \sin(\omega_0 t)$	
	10	If convolution is performed between two signals, x and h, with lengths N_x and N_h, then what will be the length N of resulting signal?	
	11	Define Invertible system.	
	12	Let $x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & otherwise \end{cases}$ and $h[n] = \begin{cases} 1, & 0 \le n \le N \\ 0, & otherwise \end{cases}$ Where N≤ 9 is an integer. Determine the value of N, given that $y[n] = x[n] * h[n]$ and $y[4] = 5$, $y[14] = 0$.	
	13	Integration of unit impulse function over $(-\infty, \infty)$ yields _____ signal and differentiating a unit ramp function yields _____ signal.	
	14	Find the even and odd components of following signal: $x(t) = 1 + t + 3t^2 + 5t^3 + 9t^4$	
Q.2	**(a)**	Categorize the following signals as an energy or power signal and find energy or power of the signal. (i) $x(t) = 5\cos(3\pi t) + \sin(5\pi t)$; $-\infty < t < \infty$ (ii) $x[n] = \begin{cases} \cos(\pi n) & ; n \ge 0 \\ 0 & ; otherwise \end{cases}$	**03**
	(b)	Consider a system S with input x[n] and output y[n] related by $y[n] = x[n](g[n] + g[n-1])$ (i) If $g[n] = 1$ for all n, show that S is time invariant. (ii) If $g[n] = n$, show that S is not time invariant. (iii)If $g[n] = 1 + (-1)^n$, show that S is time invariant.	**04**
	(c)	A continuous time signal $x(t)$ is shown in figure: Sketch and label carefully	**07**

each of the following signals.

(i) $x\left(\frac{n}{2}\right)$ **(ii)** $x(t)\left[\delta\left(t+\frac{2}{2}\right)-\delta\left(t-\frac{2}{2}\right)\right]$



OR

(c) Determine whether each of them is (i) memoryless (ii) stable (iii) causal and (iv) linear **07**

(1) $y[n]=2x[2^n]$

(2) $y(t)=x(t/2)$ Justify your answers.

Q.3 **(a)** State and prove a condition for a discrete time LTI system to be invertible. **03**

(b) The following are the impulse responses of LTI systems. Determine whether each system is causal and/or stable. Justify your answers. **04**

(i) $h[n]=(0.8)^n u[n+2]$ (ii) $h(t)=e^{2t}u(-t-2)$

(c) Find the convolution of two signals $x(t)$ and $y(t)$ **07**

$$x(t)=\begin{cases}1, & |t|<1\\ 0, & \text{otherwise}\end{cases}$$

$$y(t)=\begin{cases}1, & |t|<1\\ \delta(t+2)+\delta(t-2) & \\ 0, & \text{otherwise}\end{cases}$$

OR

Q.3 **(a)** State and prove a condition for a discrete time LTI system to be stable. **03**

(b) Evaluate the step response for the LTI systems represented by the following impulse responses: **04**

(i) $h(t)=e^{-|t|}$ **(ii)** $h[n]=(-1)^n\{u[n+2]-u[n-3]\}$

(c) Evaluate the discrete time convolution sum given below. **07**

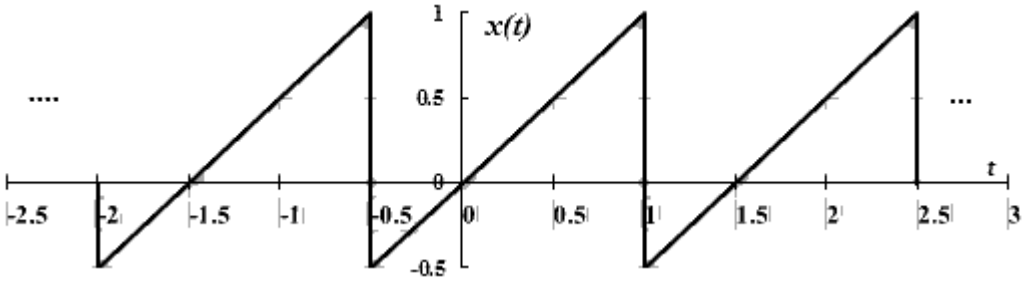
$y[n]=\beta^n u[n]*u[n-3],\quad |\beta|<1$

Q.4 **(a)** Determine the output of the system described by the following differential equation with input and initial condition as specified. **03**

$\frac{dy(t)}{dt}+10y(t)=2x(t)$, $y(0)=1$, $x(t)=u(t)$

(b) Write Differentiation in Time and Differentiation in Frequency property of Fourier Transform. Obtain Fourier Transform of $\frac{d(e^{-at}u(t))}{dt}$ **04**

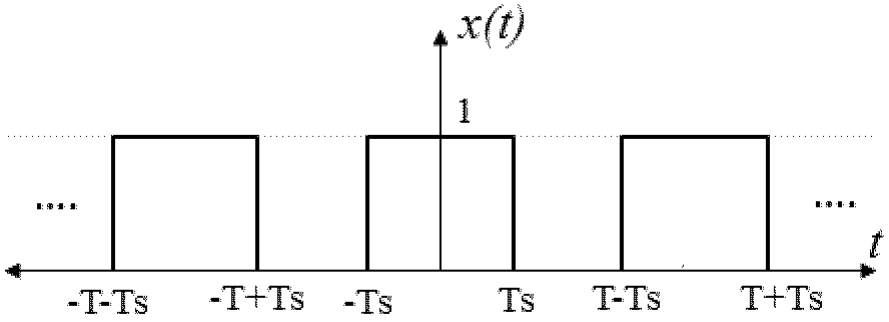
(c) Find the Fourier Series representation for the sawtooth wave depicted in below figure. **07**



OR

Q.4 **(a)** Write modulation property of Fourier Transform. Use frequency differentiation property to find the Fourier Transform of $x(t)=te^{-at}u(t)$. **03**

(b) Determine the Complex Exponential Fourier Series representation for the square wave depicted in below figure. **04**



(c) Write Duality property of Fourier transform. Given $x(t) = \begin{cases} 1, & |t| < 2 \\ 0, & |t| > 2 \end{cases}$. Find the Fourier transforms of following: **07**

(i) $e^{-a|t|}$ **(ii)** $\frac{1}{2\pi(j2\pi f+1)}$

Q.5 **(a)** Compute DFT of the following sequence: $x[n] = \{0,1,2,3\}$ **03**

(b) Determine z-transform of following sequences. **04**

(i) $x[n] = a^{|n|}, 0 < |a| < 1$

(ii) $x[n] = 2^n u[n] + 3^n u[-n-1]$.

(c) Find the z-transform of the signal **07**

$$x[n] = \left\{n\left(\frac{-1}{2}\right)^n u[n]\right\} * \left\{\left(\frac{1}{4}\right)^n u[-n]\right\}$$

OR

Q.5 **(a)** Find the impulse response $h[n]$ for each of the causal LTI discrete time systems satisfying the following difference equations. **03**

(i) $y[n] = x[n] - 2x[n-2] + x[n-3]$

(ii) $y[n] + 2y[n-1] = x[n] + x[n-1]$

(b) Write Differentiation in Z-domain property of z- transform. Obtain z-transform of $x[n] = a^n \cos(\Omega_0 n) u[n]$, where a is real and positive. **04**

(c) Find the inverse z-transform of **07**

$X(z) = \frac{z^2 - 1.08z^2 - 4z + 4}{2z^2 - 2z - 4}$ with ROC $|z| < 1$

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