

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-IV(NEW) – EXAMINATION – SUMMER 2019

Subject Code:2141005

Date:28/05/2019

Subject Name: Signals and Systems

Time:02:30 PM TO 05:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Find whether given signal is periodic or not? If yes, give its fundamental period. 03

(i) $x(t) = 3\cos(10\pi t) + 5\sin(6\pi t)$

(ii) $x[n] = e^{j10n}$

(b) Decompose following signals into their even and odd parts. 04

(i) $x(t) = 3t^2 + 2t + 1$

(ii) $x[n] = \{1, \underset{\uparrow}{1}, 1\}$

(c) Explain following property for the system $y(t) = x(t) + 2$. 07
 (i) Linearity (ii) Time-invariance (iii) Causality (iv) Dynamicity
 (v) Stability.

Q.2 (a) Let $x[n]$ be a signal with $x[n] = 0$ for $n < -2$ and $n > 4$. For each of the following signal, determine the values of n for which it is guaranteed to be zero. 03

(i) $x[n - 3]$

(ii) $x[-n - 2]$

(b) Prove associativity property of convolution sum. 04

(c) For, 07

$x[n] = \delta[n] + 2\delta[n - 1] - \delta[n - 3]$ and

$h[n] = 2\delta[n + 1] + 2\delta[n - 1]$.

Compute (i) $y_1[n] = x[n] * h[n]$ and (ii) $y_2[n] = x[n] * h[n + 2]$.

OR

(c) Sketch each of the following signals for a signal shown in Figure 1. 07

(i) $x(2 - t)$

(ii) $x(2t + 1)$

(iii) $[x(t) + x(-t)]u(t)$ [2+2+3 Marks]

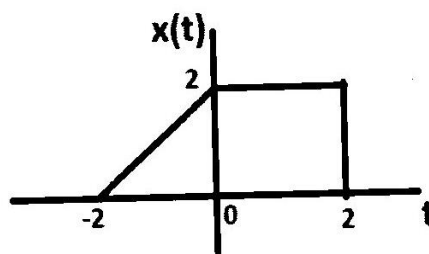


Figure 1

Q.3 (a) Find the convolution $x[n] * h[n]$, where $x[n] = \{1, 2, 3\}$ and $h[n] = \{1, 0, 1\}$. 03

↑

- (b) Given that $y[n] = x[n] * h[n]$, $x[n] = \{1, 0, 1\}$, $y[n] = 0$ for $n < -1$ and $y[n] = 2$ for $n = -1, 0, 3$. Find $h[n]$. Given $y[n]$ is of finite duration signal with length of 5. 04

- (c) Consider periodic signal $x(t)$ with fundamental frequency $\omega_0 = \pi$, determine its complex exponential Fourier series representation. Where, 07

$$x(t) = \begin{cases} 1.5, & 0 \leq t < 1 \\ -1.5, & 1 \leq t < 2 \end{cases}$$

OR

- Q.3 (a) Find the convolution $x(t) * h(t)$, where $x(t) = h(t) = e^{-at}u(t)$. 03
 (b) Given that $x[n]$ has Fourier transform $X(e^{j\omega})$, express the Fourier transform of the $w[n] = (n-1)^2x[n]$ in terms of $X(e^{j\omega})$. [hint: Use Fourier transform property.] 04
 (c) Let $x(t)$ be a periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k . Given that $y(t) = x(1-t) + x(t-1)$, how is the fundamental frequency ω_2 of $y(t)$ related to ω_1 ? Also, find a relationship between the Fourier series coefficients b_k of $y(t)$ and the coefficients a_k . 07

- Q.4 (a) State and prove Time scaling property of Fourier transform. 03
 (b) Given the relationships $y(t) = x(t) * h(t)$ and $g(t) = x(3t) * h(3t)$. Also given that $x(t)$ and $h(t)$ have Fourier transform $X(j\omega)$ and $H(j\omega)$ respectively. Using Fourier transform property show that $g(t) = Ay(Bt)$ and determine the values of A and B. 04
 (c) Find the response of an LTI system with impulse response $h(t) = e^{-at}u(t)$, $a > 0$ to the input signal $x(t) = e^{-bt}u(t)$, $b > 0, a \neq b$, using Fourier transform. 07

OR

- Q.4 (a) State and prove Duality property of Fourier transform. 03
 (b) A stable LTI system characterized by the differential equation 04

$$\frac{dy(t)}{dt} + ay(t) = x(t), \quad a > 0.$$

 Find the impulse response of the system.
 (c) Find the Fourier transform of $x(t) = e^{-|t|}$. Using property find Fourier transform of $\frac{2}{1+t^2}$. 07

- Q.5 (a) Find the inverse z-transform of $X(z) = z^{-2} + 1 + z^3, 0 < z < \infty$. 03
 (b) Discuss causality and stability of LTI system using z-transforms. 04
 (c) Using partial fraction expansion find the inverse z-transform of 07

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{(1 - z^{-1})(1 + 2z^{-1})}, \quad |z| > 2$$

OR

- Q.5 (a) Find DTFT of $x[n] = \{1, 0, 4, 2\}$. 03
 (b) State and prove differentiation property of z-transform. 04
 (c) Consider the following algebraic expression for the z-transform $X(z)$ of signal $x[n]$: 07

$$X(z) = \frac{1 + z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

- (i) Assuming the ROC to be $|z| > 1/3$, use long division to determine the values of $x[0]$, $x[1]$, and $x[2]$.
 (ii) Assuming the ROC to be $|z| < 1/3$, use long division to determine the values of $x[0]$, $x[-1]$, and $x[-2]$.
