Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER–IV (New) EXAMINATION – WINTER 2015

Subject Code:2141005 **Date:04/01/2016**
Subject Name: Signals and Systems
Time: 2:30pm to 5:00pm **Total Marks: 70**

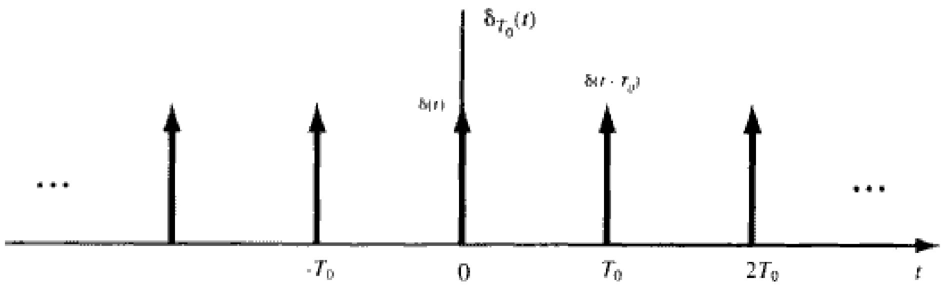
Instructions:
1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Define : Signal. **07**
Find the fundamental periods (T for continuous-time signals, N for discrete-time signals) of the following periodic signals.
1. $x(t) = \cos(13\pi t) + 2\sin(4\pi t)$
2. $x[n] = e^{j7.351\pi n}$

(b) Define: System. **07**
Determine whether the system $y(t) = t\,x(t)$ is
1. Memoryless
2. Linear
3. Time invariant
4. Causal
5. BIBO stable. Justify your answers.

Q.2 **(a)** Compute the convolution y(n) = x(n) * h(n) **07**
1. x[n] =δ[n] – δ[n-2], h[n] = u[n]
2. x[n] =u[n], h[n] = u[n]

(b) Determine the trigonometric Fourier series for signal given below $\delta_{T_0}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_0)$. **07**



OR

(b) Determine the complex exponential Fourier series for **07**
1. $\cos(\omega_0 t)$
2. $\sin^2 t$

Q.3 **(a)** Define: The continuous time Fourier transform. **07**
State and prove Time shifting and Duality properties of continuous time Fourier transform.

(b) Find the Z transform of **07**

1. $\delta(n)$
2. $u[n]$
3. $na^n u[n]$. (1+2+4 Marks)

OR

Q.3 **(a)** Define: The Z transform. **07**

State and prove Time shifting and Time reversal properties of Z transform.

(b) Find the continuous time Fourier transform

1. $\delta(t)$ **07**
2. $e^{-at}u[t], a > o$
3. $u[t]$. (1+2+4 Marks)

Q.4 **(a)** Using power series expansion technique find the inverse Z transform of **07**

$X(z) = \frac{1}{1-az^{-1}}, |z| > |a|$.

(b) The output $y[n]$ of a discrete-time LTI system is found to be $2\left(\frac{1}{3}\right)^n u[n]$ when the input $x[n]$ is $u[n]$. Find the impulse response $h[n]$ of the system. **07**

OR

Q.4 **(a)** Using the partial fraction expansion technique find the inverse Z transform of **07**

$X(Z) = \frac{z}{2z^2 - 3z + 1}, \ |z| < \frac{1}{2}$.

(b) For the differential equation $y[n] - \frac{1}{2} y[n-1] = x[n]$ with input $x[n] = \left(\frac{1}{3}\right)^n$ and for initially $y[-1] = 1$ find the output $y[n]$. **07**

Q.5 **(a)** Define: Convolution Sum. **07**

Show that

1. $x[n] * \delta[n] = x[n]$
2. $x[n] * \delta[n - n_0] = x[n - n_0]$
3. $x[n] * u[n - n_0] = \sum_{k=-\infty}^{n-n_0} x[k]$

(b) A Continuous-time periodic signal *x(t)* is real valued and has a fundamental period T = 8. The nonzero Fourier series coefficients for *x(t)* are **07**

$$a_1 = a_{-1} = 2, a_3 = a_{-3}^* = 4j.$$

Express *x(t)* in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

OR

Q.5 **(a)** Define the condition for LTI system to be stable. Which of the following impulse responses correspond to stable LTI systems. **07**

1. $h_1(t) = e^{-(1-2j)t} u(t)$
2. $h_2(n) = 3^n u[-n+10]$

(b) Define Laplace transform. Prove linearity property for Laplace transform. State how ROC of Laplace transform is useful in defining stability of systems. **07**

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