

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-VI (NEW) - EXAMINATION – SUMMER 2018

Subject Code:2160704

Date:03/05/2018

Subject Name:Theory of Computation

Time:10:30 AM to 01:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

Q.1 (a) Show that the CFG with productions $S \rightarrow a \mid Sa \mid bSS \mid SSb \mid SbS$ is ambiguous. **03**

(b) Define onto function. In each case, a relation on the set $\{1, 2, 3\}$ is given. Of the three properties, reflexivity, symmetry, and transitivity, determine which ones the relation has. Give reasons. **04**

- a. $R = \{(1, 3), (3, 1), (2, 2)\}$
- b. $R = \{(1, 1), (2, 2), (3, 3), (1, 2)\}$
- c. $R = \phi$

(c) Write Principle of Mathematical Induction. Prove that for every $n \geq 1$, $\sum_{i=1}^n \frac{1}{i(i+1)} = n / (n+1)$ **07**

Q.2 (a) Explain Chomsky Hierarchy. **03**

(b) Convert the given Moore machine into Mealy machine. Draw state transition diagram of Mealy machine. **04**

Present State	Next State		Output
	0	1	
$\rightarrow p_0$	r	q ₀	ϵ
p ₁	r	q ₀	1
q ₀	p ₁	s ₀	0
q ₁	p ₁	s ₀	1
r	q ₁	p ₁	0
s ₀	s ₁	r	0
s ₁	s ₁	r	1

(c) Given the context-free grammar G, find a CFG G' in Chomsky Normal Form. **07**

G :

$$\begin{aligned} S &\rightarrow AaA \mid CA \mid BaB \\ A &\rightarrow aaBa \mid CDA \mid aa \mid DC \\ B &\rightarrow bB \mid bAB \mid bb \mid aS \\ C &\rightarrow Ca \mid bC \mid D \\ D &\rightarrow bD \mid \epsilon \end{aligned}$$

ϵ represents null.

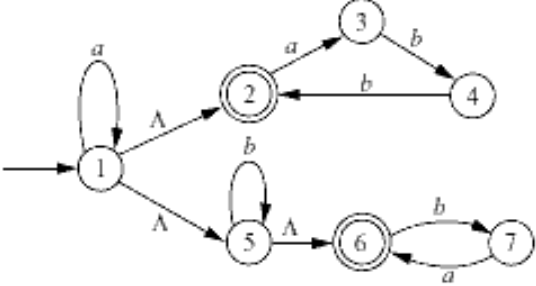
OR

(c) Define Context Free Grammar. Find context-free grammar for the language: $L = \{a^i b^j \mid i < 2j\}$ **07**

Q.3 (a) Show that the function $f(x, y) = x + y$ is primitive recursive. **03**

(b) Explain Union Rule and Concatenation Rule for Context-Free Grammar. **04**

(c) Figure shows NFA- Λ . Draw an FA accepting the same language. 07



OR

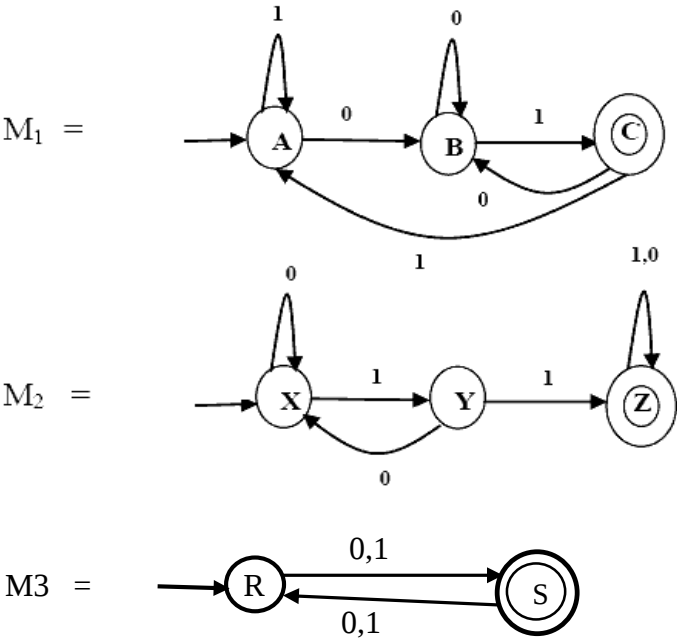
Q.3 (a) Define Constant functions, Successor functions and Projection function. 03

(b) Let G be the grammar 04

$$\begin{aligned} S &\rightarrow aB \mid bA \\ A &\rightarrow a \mid aS \mid bAA \\ B &\rightarrow b \mid bS \mid aBB \end{aligned}$$

For string aaabbabbba, find Left most derivation and Right most derivation.

(c) Let M_1 , M_2 and M_3 be the FAs pictured in Figure, recognizing languages L_1 , L_2 and L_3 , respectively. 07



Draw FAs recognizing the following languages.

- a. $L_1 \cup L_2$
- b. $L_1 \cap L_3$

Q.4 (a) Decide whether the given language is a CFL, and prove your answer. 03

$$L = \{ xyx \mid x, y \in \{a, b\}^* \text{ and } |x| \geq 1 \}$$

(b) Construct PDA for 04

$$\begin{aligned} S &\rightarrow 0AB \\ A &\rightarrow 1A \mid 1 \\ B &\rightarrow 0B \mid 1A \mid 0 \end{aligned}$$

Trace the string 01011 using PDA.

(c) Give transition tables for deterministic PDA recognizing following language: 07

$$L = \{ x \in \{a, b\}^* \mid n_a(x) \neq n_b(x) \}$$

Trace it for the string abbaababbb

OR

Q.4 (a) Show using pumping lemma that the given language is not a CFL. **03**

$$L = \{ a^n b^{2n} a^n \mid n \geq 0 \}$$

(b) Prove that There are CFLs L_1 and L_2 so that $L_1 \cap L_2$ is not a CFL, and there is a CFL L so that L' is not a CFL. **04**

(c) For the PDA, $(\{q_0, q_1\}, \{0, 1\}, \{0, 1, z_0\}, \delta, q_0, z_0, \phi)$, **07**
where δ is

$$\begin{aligned}\delta(q_0, \epsilon, z_0) &= \{(q_1, \epsilon)\} \\ \delta(q_0, 0, z_0) &= \{(q_0, 0z_0)\} \\ \delta(q_0, 0, 0) &= \{(q_0, 00)\} \\ \delta(q_0, 1, 0) &= \{(q_0, 10)\} \\ \delta(q_0, 1, 1) &= \{(q_0, 11)\} \\ \delta(q_0, 0, 1) &= \{(q_1, \epsilon)\} \\ \delta(q_1, 0, 1) &= \{(q_1, \epsilon)\} \\ \delta(q_1, 0, 0) &= \{(q_1, \epsilon)\} \\ \delta(q_1, \epsilon, z_0) &= \{(q_1, \epsilon)\}\end{aligned}$$

Obtain CFG accepted by the above PDA.

Q.5 (a) Find a regular expression corresponding to each of the following subsets of $\{0, 1\}^*$ **03**

1. The language of all strings that begin or end with 00 or 11.
2. The language of all strings containing both 11 and 010 as substrings.

(b) Define Context-Sensitive Grammar. Write a CSG for $\{a^n b^n c^n \mid n \geq 1\}$. **04**

(c) Draw a transition diagram for a Turing machine for the language of all palindromes over $\{a, b\}$. **07**

OR

Q.5 (a) Use the pumping lemma to show that following language is not regular. **03**
 $L = \{xy \mid x, y \in \{0, 1\}^* \text{ and } y \text{ is either } x \text{ or } x^r\}$

(b) Write Short note on Church-Turing Thesis. **04**

(c) Draw a transition diagram for a Turing machine accepting the language $\{SS \mid S \in \{a, b\}^*\}$. **07**
