GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-VII (NEW) EXAMINATION - WINTER 2018

Subject Code: 2171901 Date: 26/11/2018

Subject Name: Operation Research

Time: 10:30 AM TO 01:00 PM Total Marks: 70

Instructions:

- 1. Attempt all questions.
- 2. Make suitable assumptions wherever necessary.
- 3. Figures to the right indicate full marks.
- Q.1 (a) Illustrate graphically for Linear Programming Problem;

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- (a) No-feasible solution (b) Unbounded solution.
- (b) A firm plans to purchase atleast 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain atleast 100 quintals of X-metal and not more than 35 quintals of Y-metal. The firm can purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of X and Y metals in terms of weight in the scraps supplied by A and B is given below:

| Metals | Suppliers A | Suppliers B |
|--------|-------------|-------------|
| X | 25% | 75% |
| Y | 10% | 20% |

The price of supplier A's scrap is Rs. 200 per quintal and that of supplier B's is Rs. 400 per quintal. Formulate this problem as LP model to minimize total purchase cost.

(c) Solve by Simplex method the following L.P. problem

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Minimize
$$Z = 500 x_1 + 200 x_2$$

Subjected to the constraints,

$$3 x_1 + 2 x_2 \le 90$$

$$x_1 \ge 10$$

$$x_2 = 2 x_1$$

$$x_1, x_2 \ge 0$$

Q.2 (a) Discuss in brief the areas of application of Operation Research.

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(b) Find the dual problem for the following:

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Minimize
$$Z = 5x_1 - 6x_2 + 4x_3$$

Subject to the constraints,

$$3x_1 + 4x_2 + 6x_3 \ge 9$$

$$x_1 + 3x_2 + 2x_3 \ge 5$$
$$7x_1 - 2x_2 - x_3 \le 10$$

$$x_1 - 2x_2 + 4x_3 \ge 4$$

$$2x_1 + 5x_2 - 3x_3 = 3$$

$$x_1, x_2, x_3 \ge 0$$

(c) The data for a network is given below.

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| Activity | Time i | n days | Direct cost (Rs.) | | |
|----------|--------|--------|-------------------|-------|--|
| | Normal | Crash | Normal | Crash | |
| 1-2 | 10 | 4 | 2300 | 3100 | |
| 1-3 | 12 | 3 | 1700 | 2600 | |
| 2-4 | 20 | 10 | 2400 | 4400 | |
| 3-5 | 10 | 9 | 1100 | 1500 | |
| 4-5 | 6 | 6 | 800 | 800 | |

Considering indirect cost as Rs. 1000 per/day.

- (i) Find the normal duration of project completion and corresponding cost.
- (ii) What can be minimum compression of project and corresponding total time?
- (iii) Find the optimal duration of project completion and related cost.

A construction company is preparing a PERT network for laying the foundation of a new art museum. Given the following set of activities, their predecessor requirements and three time estimates of completion time:

| | | Time in weeks | | | |
|------------|--------------|---------------|-------------|--------|--|
| Activities | Predecessors | Optimistic | Pessimistic | Most | |
| | NT | 2 | 4 | Likely | |
| A | None | 2 | 4 | 3 | |
| В | None | 8 | 8 | 8 | |
| С | A | 7 | 11 | 9 | |
| D | В | 6 | 6 | 6 | |
| Е | С | 9 | 11 | 10 | |
| F | С | 10 | 18 | 14 | |
| G | C, D | 11 | 11 | 11 | |
| Н | F, G | 6 | 14 | 10 | |
| I | Е | 4 | 6 | 5 | |
| J | I | 3 | 5 | 4 | |
| K | Н | 1 | 1 | 1 | |

- (a) Draw the PERT network.
- (b) What is the expected time of the duration of the project?
- (c) What is the probability that the project will be completed in 40 weeks?

Use the following data:

| Z 1.0 | | 1.22 | 1.57 | 1.87 |
|-------------|--------|--------|--------|--------|
| Probability | 0.8413 | 0.8888 | 0.9418 | 0.9693 |

- Q.3 Define the following terms: balking, reneging and jockeying. (a)
 - Explain in brief characteristic of queuing theory. What is traffic intensity? If
 - traffic intensity of a system is given to be 0.76, what percent of time the system would be idle?
 - The following is the pay-off matrix between player X and player Y. Find the (c) optimal strategies, their frequencies and the value of the game. Use rule of dominance and oddment in calculations.

Player 2

| | | A | В | C | D |
|---|-----|-------|------|------|------|
| X | I | 0.25 | 0.20 | 0.14 | 0.30 |
| Λ | II | 0.27 | 0.16 | 0.12 | 0.14 |
| | III | 0.35 | 0.08 | 0.15 | 0.19 |
| | IV | -0.02 | 0.08 | 0.13 | 0.00 |

Player Y

OR

- Q.3 What do you understand by 'zero-sum' in the context of game theory? Explain the meaning following terms used in game theory;
 - i. Saddle Point
 - ii. Pure Strategy
 - iii. Mixed Strategy

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- (b) Arrival rate of telephone calls at a telephone booth is according to Poisson distribution, with an average time of 9 minutes between consecutive arrivals. The length of telephone call is exponentially distributed with a man of 3 minutes. Find:
 - i. Determine the probability that a person arriving at the booth will have to wait.
 - ii. Find the average queue length that forms from time to time.
 - iii. The telephone company will install a second booth when conveniences that an arrival would expect to have to wait at least four minutes for the phone. Find the increase in flow of arrivals, which will justify a second booth.
 - iv. What is the probability that an arrival will have to wait for more than 10 minutes before the phone is free?
 - v. Find the fraction of a day that the phone will be in use.
- Q.4 (a) Explain the meaning of following items in inventory management;

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- a. Re-order Level
- b. Buffer Stock
- (b) Discuss the similarity and differences between PERT and CPM.

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(c) A stockiest of a particular commodity makes a profit of Rs. 30 on each sale made within the same week of purchase; otherwise he incurs a loss of Rs. 30 on each item. The data on the past sales are given below:

| No. of items sold within the same week | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--|---|---|----|----|---|----|----|
| Frequency | 0 | 9 | 12 | 24 | 9 | 6 | 0 |

- i. Find out the optimum number of items the stockiest should buy every week in order to maximize the profit.
- ii. Calculate the expected value of perfect information.

OR

Q.4 (a) Explain ABC analysis.

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- (b) The truck-owner finds from his past experience that the maintenance costs are Rs. 200 for the first year and then increase by Rs. 2000 every year. The cost of truck type A is Rs. 9000. Determine the best age at which to replace the truck. If the optimum replacement is followed what will be the average yearly cost of owning and operating the truck? Truck of type B costs Rs. 20000. Annual operating costs are Rs. 400 for the first year and then increase by Rs. 800 every year. The truck owner has now the truck type A which is one year old. Should it be replaced by B type truck, and if so when?
- Q.5 (a) A hardware store procures and sells hardware items. Information on an item is given here:

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Expected annual sales = 8000 units

Ordering cost = Rs. 180 per order

Holding cost = 10% of the average inventory value

The item can be purchased according to the following schedule:

| Lot size | Unit price (Rs.) |
|----------------|------------------|
| 1-999 | 22 |
| 1000-1499 | 20 |
| 1500-1999 | 19 |
| 2000 and above | 18.5 |

Determine the best order size.

(b) Solve the following transportation problem for maximum profit. Use vogel's approximation method to find out initial feasible solution.

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Market (Per unit profit in Rs.)

| | | | 1 | | | | |
|-----------|---|----|----|----|----|--|--|
| | | A | В | C | D | | |
| | X | 12 | 18 | 6 | 25 | | |
| Warehouse | Y | 8 | 7 | 10 | 18 | | |
| | Z | 14 | 3 | 11 | 20 | | |

| Availability in warehouses | Demand in the markets |
|----------------------------|-----------------------|
| X = 200 units | A = 180 units |
| Y = 500 units | B = 320 units |
| Z = 300 units | C = 100 units |
| | D = 400 units |

OR

What is degeneracy in transportation problems? Explain how to resolve **Q.5** (a) degeneracy in a transportation problem.

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You are given the information about the cost of performing different jobs by different persons. The job person making 'X' indicates that the individual involved cannot perform the particular job. Using this information, state (i) the optimal assignment of job. (ii) the cost of such assignment.

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Person

| | J_1 | J_2 | J ₃ | J ₄ | J_5 |
|----------------|-------|-------|----------------|----------------|-------|
| \mathbf{P}_1 | 27 | 18 | X | 20 | 21 |
| \mathbf{P}_2 | 31 | 24 | 21 | 12 | 17 |
| P 3 | 20 | 17 | 20 | X | 16 |
| P ₄ | 22 | 28 | 20 | 16 | 27 |

Job
