

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-VIII (NEW) - EXAMINATION – SUMMER 2018

Subject Code: 2181911

Date: 30/04/2018

Subject Name: Finite Elements Method(Department Elective II)

Time: 10:30 AM to 01:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1**
- | | | |
|-----|--|-----------|
| (a) | Can the FEM handle a wide range of problems, i.e., solve general PDEs? Enlist two advantages of FEM. | 03 |
| (b) | List four applications of FEM and computer programs used for the FEM. | 04 |
| (c) | List and briefly describe the process of the Finite Element Method. | 07 |
- Q.2**
- | | | |
|-----|---|-----------|
| (a) | What are the characteristics of shape function? Why polynomials are generally used as shape function? | 03 |
| (b) | Draw three 2D and 3D types of finite element. | 04 |
| (c) | Derive the Stiffness Matrix for a Spring Element. | 07 |
- OR**
- (c) (a) Formulate the global stiffness matrix and equations for solution of the unknown global displacement and forces. The spring constants for the elements are k_1 , k_2 , and k_3 ; P is an applied force at node 2. **07**
(b) Using the direct stiffness method, formulate the same global stiffness matrix and equation as in part (a).

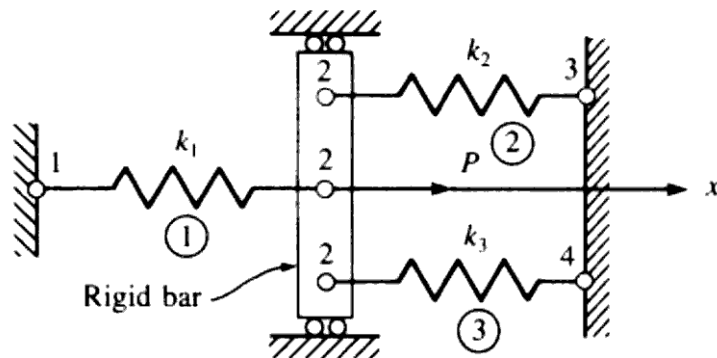


Figure 1

- Q.3**
- | | | |
|-----|--|-----------|
| (a) | Distinguish between essential boundary conditions and natural boundary conditions. Give their examples. | 03 |
| (b) | Discuss the penalty approach for FEM. | 04 |
| (c) | A tapered bar 1200 mm long, having cross-sectional area 450 mm ² at one end and 150 mm ² at other end is fixed at the larger end. It is subjected to an axial load of 35 kN. Calculate the stress on a model bar having three finite elements 400 mm long. Assume modulus of elasticity, $E = 2 \times 10^5$ N/mm ² circular cross section at both end. | 07 |

OR

- Q.3 (a)** For the loading system as shown in Figure 2, determine the element stiffness matrix and global stiffness matrix. Assume modulus of elasticity as $80 \times 10^3 \text{ N/mm}^2$ **03**

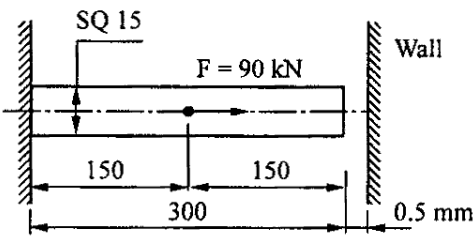


Figure 2

- (b)** For above Q. 3 (a) determine the displacements, stresses and support reaction using penalty approach. **04**
- (c)** Axial load $P = 300 \text{ kN}$ is applied at 20° C to the rod as shown in Figure 3. The temperature is then raised to 60° C . The coefficient of thermal expansion for Aluminium is 23×10^{-6} per $^\circ \text{C}$ and Steel is 11.7×10^{-6} per $^\circ \text{C}$. $A_{\text{Al}} = 900 \text{ mm}^2$, $A_{\text{Steel}} = 1200 \text{ mm}^2$, $E_{\text{Al}} = 70 \times 10^9 \text{ N/m}^2$, $E_{\text{Steel}} = 200 \times 10^9 \text{ N/m}^2$. Using FEM, Determine the nodal displacement and element stresses and the reaction forces at the supports. **07**

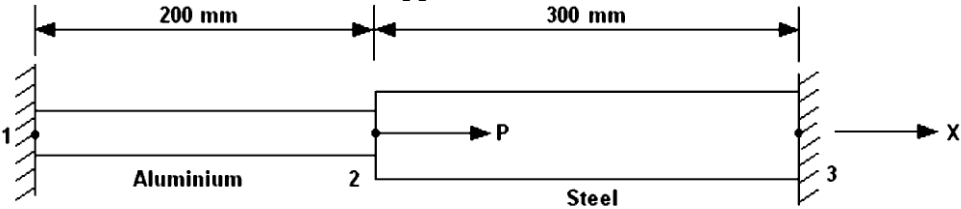


Figure 3

- Q.4 (a)** Write the shape function and stiffness matrix for one-dimensional finite element formulation of the fluid-flow problem. **03**
- (b)** Derive the element stiffness matrix of truss element and write the stress calculation formula for truss. **04**
- (c)** A three bar truss is shown in Figure 4. The modulus of elasticity of the material is $300 \times 10^3 \text{ N/mm}^2$. The area of the bar used for the truss is 60 mm^2 for all the elements. The length $L_1 = 750 \text{ mm}$ and $L_2 = 100 \text{ mm}$. The load $P_1 = 20 \text{ kN}$ and $P_2 = 25 \text{ kN}$. Determine the element stiffness matrix for each element and the global stiffness matrix. **07**

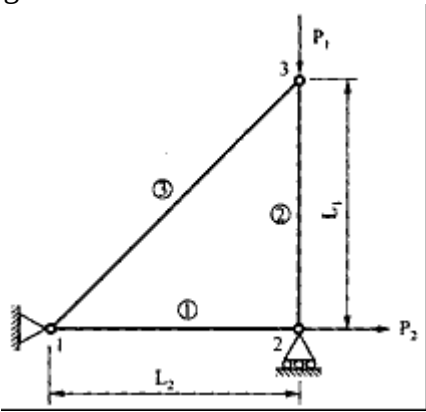


Figure 4

OR

- Q.4

(a)

List out the application of axisymmetric elements.

03

(b)

Discuss the terms “plain stress” and “plain strain” problems.

04

(c)

Evaluate the stiffness matrix for the element shown in Figure 5. The coordinates are shown in units of inches. Assume plane stress conditions. Let $E = 30 \times 10^6$ psi, $\nu = 0.25$, and thickness $t = 1$ in. Assume the element nodal displacements have been determined to be $u_1 = 0$, $v_1 = 0.0025$ in., $u_2 = 0.0012$ in., $v_2 = 0$, $u_3 = 0$, and $v_3 = 0.0025$ in. Determine the element stresses.

07

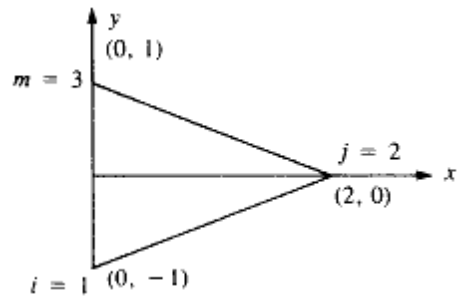


Figure 5 Plane stress element for stiffness matrix evaluation

- Q.5

(a)

Write the point force, body force and surface traction force using natural coordinate system.

03

(b)

Write the four shape function equations for a beam element.

04

(c)

Using the direct stiffness method, solve the problem of the propped cantilever beam subjected to end load P in Figure 6. The beam is assumed to have constant EI and length 2L. It is supported by a roller at mid length and is built in at the right end. Propped cantilever beam shown in below Figure 6

07

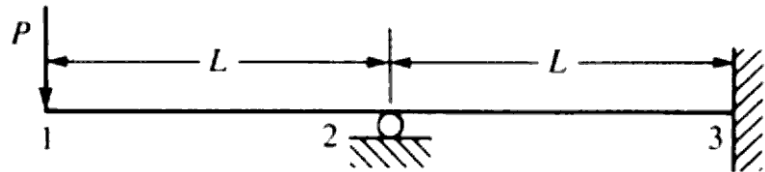


Figure 6
OR

- Q.5

(a)

Write the consistent and lumped mass matrices for 1D element.

03

(b)

List out applications of the axisymmetric elements.

04

(c)

For the smooth pipe shown discretized in Figure 7 with uniform cross section of 1 in², determine the flow velocities at the center and right end, knowing the velocity at the left end is $v_x = 2$ in./s.

07

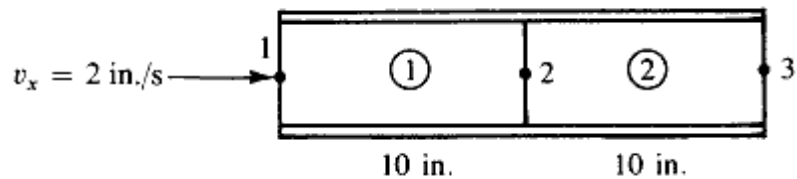


Figure 7 Discretized pipe for fluid-flow problem
